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MATC01: GROUPS AND SYMMETRY

Problem Set 2 – due Friday, September 27th

INSTRUCTIONS:
You (personally) must turn this in during the first 5 minutes of lecture on the due date. Please print and attach this page as the first page of your submitted problem set.

PROBLEM	**MARK**
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Please read the following statement and sign below:

I understand that I am not allowed to use the internet to assist (in any way) with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person.

SIGNATURE:____________________

Problem Set 2

2.1 Given $A \in \mathbb{R}^2$ such that $A \cdot X = 0$ for all $X \in \mathbb{R}^2$. Prove that $A = (0, 0)$. [*Here the operation $\cdot$ is the vector dot product.*]

2.2 Prove that every isometry of the form $T_h \circ R_\alpha$ with $\alpha \neq 0$ is a rotation. [*This is the converse of the statement we proved in class.*]

2.3 Suppose that m is an isometry which satisfies $m(0) = 0$. Prove that for all $w, z \in \mathbb{C}$, we have

$$m(w + z) = m(w) + m(z).$$

For the next three problems, we introduce three new terms to describe isometries: *primitive*, *orientation-preserving*, and *orientation-reversing*. We say $\phi \in G$ is *primitive* iff ϕ is one of T_h, R_α, or ρ. (For example, T_{2-3i} is a primitive isometry, while $T_{2-3i} \circ R_{\pi/3}$ is not.) At the end of class on Friday, September 20th, we stated a lemma that every isometry is a composition of primitive isometries. More precisely, the lemma asserts that given any $\phi \in G$, there exist $h \in \mathbb{C}$, $\alpha \in \mathbb{R}$, and $j \in \{0, 1\}$ such that

$$\phi = T_h \circ R_\alpha \circ \rho^j.$$

If $j = 0$, we say ϕ is *orientation-preserving*; if $j = 1$, we say ϕ is *orientation-reversing*. For the problems below, you may assume the lemma is true. (We will prove it this week.)

2.4 Prove that every isometry is either orientation-preserving or orientation-reversing, but not both.

2.5 Prove that a composition of (finitely many) primitive isometries is orientation-preserving iff it has an even number of ρ's in it.

2.6 Recall that $\sigma_{\mathcal{L}}$ denotes the isometry which reflects across the line $\mathcal{L}$. Suppose $\mathcal{L}$ and $\mathcal{L}'$ are two lines in the plane. What can you say about the isometry $\sigma_{\mathcal{L}} \circ \sigma_{\mathcal{L}'}$? Try to be as specific as possible. Prove whatever you can.