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MATC01: GROUPS AND SYMMETRY

Problem Set 3 – due Friday, October 4th

INSTRUCTIONS:
To receive credit, you must turn this in during the first 5 minutes of lecture on the due date. Please print and attach this page as the first page of your submitted problem set.

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Please read the following statement and sign below:

I understand that I am not allowed to use the internet to assist (in any way) with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person.

SIGNATURE:________________________________

Problem Set 3

3.1 Prove that $R_\theta \rho$ is a reflection across a line. Which line?

3.2 Suppose that $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear map such that for all $X \in \mathbb{R}^2$, we have $|f(X)| = |X|$.

(a) Prove that f is either a rotation or a reflection.

(b) Prove that every eigenvalue of f is ± 1. [*Recall that an* eigenvalue *of f is any $\lambda \in \mathbb{R}$ such that $f(X) = \lambda X$ for some $X \in \mathbb{R}^2$.*]

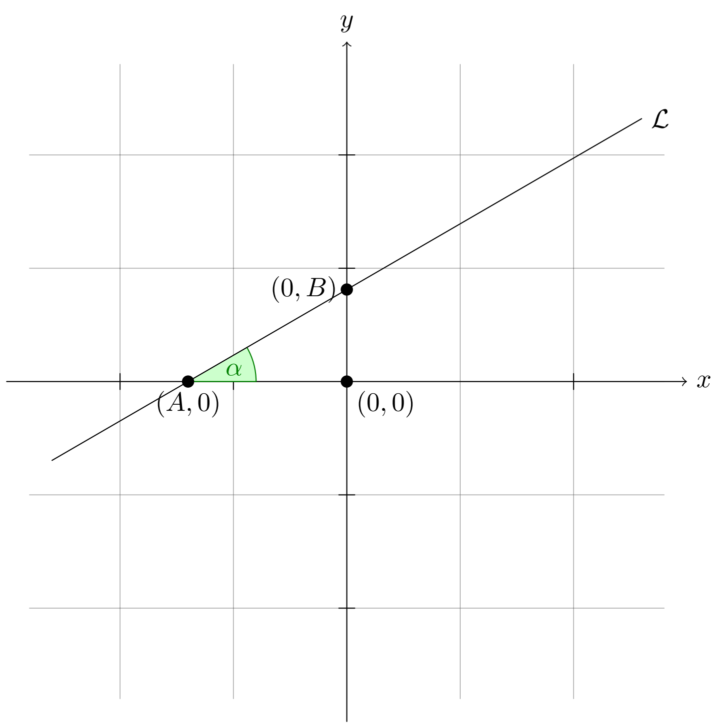
3.3 Suppose $\phi \in G$ is orientation-reversing. Prove that ϕ^2 is a translation.

3.4 Suppose $\phi \in G$. We say ϕ *fixes the point* $X \in \mathbb{R}^2$ iff $\phi(X) = X$; more generally, we say ϕ fixes the set $S \subseteq \mathbb{R}^2$ iff $\phi(S) = S$. How many points are fixed by a translation? By a rotation? By a glide reflection? How many lines are fixed by each of these? Prove whatever you can. You may assume the Classification Theorem.

3.5 Given a line $\mathcal{L}$ in the plane which makes an angle of $\alpha \in [0, \pi)$ with the x-axis. Denote the x-intercept by A (if the line intersects the x-axis) and the y-intercept by B (if the line intersects the y-axis). See the figure below.

(a) Write down an explicit isometry ϕ such that $\phi(\mathcal{L})$ is the x-axis. (Write ϕ in the form $T_h R_\theta$.)

(b) Recall from lecture that $\gamma_{\mathcal{L},a}$ denotes the glide reflection with respect to the line $\mathcal{L}$ and glide $a \in \mathbb{R}$. ($a > 0$ means the glide is up / to the right, $a < 0$ means the glide is down / to the left, and $a = 0$ means there is no glide.) Using the notation of the picture, suppose $A = -2$ and $B = 2\sqrt{3}$. Determine $h \in \mathbb{C}, \theta \in [0, 2\pi)$, and $j \in \{0, 1\}$ such that $\gamma_{\mathcal{L},-2} = T_h R_\theta \rho^j$.



The line $\mathcal{L}$ has x-intercept A and y-intercept B, and forms an angle α with the x-axis; note that we may assume that $\alpha \in [0, \pi)$.