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MATC01: GROUPS AND SYMMETRY

Problem Set 5 – due Monday, October 21st

INSTRUCTIONS:
To receive credit, you must turn this in during the first 5 minutes of lecture on the due date. Please print and attach this page as the first page of your submitted problem set.

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Please read the following statement and sign below:

I understand that I am not allowed to use the internet to assist (in any way) with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person.

SIGNATURE:________________________________

Problem Set 5

5.1 Determine whether each of the following is an equivalence relation on $\mathcal{G}$, the set of all plane isometries. If it is, prove it. If not, show (by example) all the necessary properties it fails to satisfy.

(a) (Quentin) $f \approx g$ iff there exist nonzero integers m and n such that $f^m = g^n$.

(b) (Jay) $f \approx g$ iff there exists $k \in \mathbb{R}^2$ such that $f = T_k g$.

(c) (Eric) $f \approx g$ iff f and g fix the same number of points in $\mathbb{R}^2$.

Recall that in class we defined an equivalence relation by $f \sim g$ iff there exists $\phi \in \mathcal{G}$ such that $\phi^{-1} f \phi = g$.

5.2 Let $\sim$ denote the equivalence relation define above.

(a) Suppose $f, g \in \mathcal{G}$ are both rotations by the same angle α, but around different points. Prove that $f \sim g$.

(b) Suppose $f, g \in \mathcal{G}$ are both reflections, but across different lines. Prove that $f \sim g$.

(c) Prove that if $f \sim g$, then $f^2 \sim g^2$.

(d) Suppose $R_\alpha \sim R_\beta$ for some $\alpha, \beta \in (-\pi, \pi]$. Prove that $\alpha = \pm\beta$.

5.3 Let $\sim$ denote the equivalence relation define above. Does it distinguish between translations and rotations? In other words, does there exist a nontrivial translation f and a nontrivial rotation g such that $f \sim g$? Either give an example of such isometries (with proofs!), or else prove that no such isometries exist.

5.4 (Challenge question!) Does there exist an equivalence relation on $\mathcal{G}$ such that any two nontrivial rotations are equivalent and any two nontrivial translations are equivalent, but no nontrivial rotation is equivalent to a nontrivial translation?