Instructor: Leo Goldmakher

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

Problem Set 3 (due November 1, 2010 at the start of lecture)

Please review the policies listed in the syllabus.

3.1 On the bottom half of page 83, Eves describes the notion of commensurability. On page 84, he gives a geometric proof of the irrationality of $\sqrt{2}$ in terms of this notion. Rewrite his proof, from start to finish, in your own words. As you do this, fill in various missing details in his argument (i.e. when he says "One may easily prove...", you should actually show how it's done!).

3.2 Show that there must exist irrational numbers α and β such that α^β is rational. Note that you do not have to actually find such numbers – simply show that such numbers must exist. (*Hint: you may find it helpful to start by considering* $\alpha = \beta = \sqrt{2}$.)

3.3 In lecture, I sketched a proof that the list of the five platonic solids is complete – there do not exist any other regular polyhedra. Write out a complete and thorough argument, using your own words. *Do not copy directly from any written notes when you write up your solution!*

3.4 What's wrong with the following "proof"? Follow along on Figure 3.4.

<u>Claim:</u> Every triangle is isosceles.

<u>Proof:</u>

Suppose ABC is an arbitrary triangle; a typical example is depicted in Figure 3.4. Let Z be the midpoint of the line segment $\overline{AB}$, and draw the line perpendicular to $\overline{AB}$ passing through the point Z. Find where this line intersects the line bisecting angle C; label the point of intersection by O. Draw in line segments $\overline{OA}$ and $\overline{OB}$, and drop perpendicular line segments from O to $\overline{AC}$ and $\overline{BC}$; call the points of intersection Y and X, respectively. We now argue as follows:

Step 1: $OXC \cong OYC$. This is because they have two angles the same, so they must be similar triangles. But they also have the same length hypotenuse $\overline{OC}$, so they must be congruent triangles.

Step 2: $OZA \cong OZB$. This is by SAS: $\overline{AZ} \cong \overline{BZ}$, $\overline{OZ} \cong \overline{OZ}$, and the angles between them are both right.

Step 3: $\overline{AY} \cong \overline{BX}$. This follows from Steps 1 and 2 by the Pythagorean theorem: from Step 1, $\overline{OX} \cong \overline{OY}$, and from Step 2, $\overline{OA} \cong \overline{OB}$. Since both triangles are right triangles, the third sides must be equal by the Pythagorean theorem.

Step 4: $\overline{AC} \cong \overline{BC}$. From Step 1, we know that $\overline{CY} \cong \overline{CX}$. From Step 3, we know $\overline{AY} \cong \overline{BX}$. Adding these together, we conclude that $\overline{AC} \cong \overline{BC}$. In other words, the arbitrary triangle ABC has to be isosceles!

"QED"

3.5 Do problem 3.5 (c) of Eves, on page 95. (You may have to read the preceding part of the problem.)

3.6 Explain how to achieve the following constructions, using only compass and straightedge.

(a) Construct a line segment precisely three times as long as a given line segment.

(b) Construct a line segment precisely 1/3 the length of a given line segment.

(c) Given a line $\mathcal{L}$ and a point P not on the line, construct a line through P which is parallel to $\mathcal{L}$.

(d) Given an angle, construct a line which bisects the angle (i.e. which splits the angle into two equal angles).

(e) Construct a regular pentagon whose side has the same length as a given segment.

(f) Construct a $60°$ angle.

(g) Given a segment of length 1, construct a segment of length $\sqrt{3} + 4/5$.