Kyle Vilanova

100/100 +10

Lauren Zachary
Section 103-02, Miller

Calculus I Review Sheet

Definition of the Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Definition of Continuity

A function is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$

and the $\lim_{x \to a^+}$ must equal the $\lim_{x \to a^-}$.

Intermediate Value Theorem

f is continuous on the closed interval $[a, b]$
C is any number between $f(a)$ and $f(b)$ so
c is between (a, b) so that $f(c) = C$

Mean Value Theorem

f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b)

Then there is at least one number (c) in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rolle's Theorem

f is continuous on the closed interval $[a,b]$

f is differentiable on the open interval (a,b)

If $\underline{f(a)=f(b)}$ then there is some $\underline{(c)}$ in $\underline{(a,b)}$ such that $\underline{f'(c)=0}$

First Derivative Test

$\boxed{\begin{array}{l} f'(x)=(+) \text{ fn. increasing} \quad f'(x)=(-) \text{ fn. decreasing} \\ f'(a) \quad [illegible] \quad (a) \text{ is a critical point (cp)} \end{array}}$

Let $f'(a)=0$ Then f has a local max if $f'(x)$ is pos. slightly to the left of (a) and neg. slightly to the right of (a)

$f'(x) \quad + + + \; 0 \; - - -$

f has a local min if $f'(x)$ is neg. slightly to the left of (a) and pos. slightly to the right of (a) $\quad f'(x) \quad - - - \; 0 \; + + +$

Second Derivative Test

$\boxed{f''(a)=0 \text{ th[illegible] a point of inflection of } f}$

Let $\cancel{f''(a)=0}$ $f'(a)=0$. Then

If $f''(a)>0$ then f has a local min at (a)

If $f''(a)<0$ then f has a local max at (a)

L' Hopital's Rule

If $f(a)=g(a)=0$ or ∞ then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Fundamental Theorem of Calculus

f is continuous on the closed interval $[a, b]$

$F(x)$ is any antiderivative for $f(x)$ so that $f'(x) = f(x)$

Then $\int_b^a f(x)\,dx = F(b) - F(a)$ where LHS denotes the area under the curve $y = f(x)$ from $x = a$ to $x = b$

Squeeze Theorem

$$|x|^c \sin(1/x)$$

$$-1 \leq \sin(1/x) \leq 1$$

$$-|x|^c \leq \sin(\tfrac{1}{x}) \leq |x|^c$$

$$\to 0 \leftarrow$$

Sum Rule

$h(x) = f(x) + g(x) \qquad h'(x) = f'(x) + g'(x)$

Constant Rule

$h(x) = af(x) \qquad h'(x) = af'(x)$

Product Rule

$h(x) = f(x)g(x) \qquad h'(x) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule

$h(x) = \frac{f(x)}{g(x)} \qquad h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Chain Rule

$h(x) = g(f(x)) \qquad h'(x) = g'(f(x)) \cdot f'(x)$

$h(x) = (f(x))^n \qquad h'(x) = n(f(x))^{n-1} \cdot f'(x)$

Multiple Rule

$$h(x) = f(ax) \qquad h'(x) = af'(ax)$$

Reciprocal Rule

$$h(x) = f(x)^{-1} \qquad h'(x) = -f'(x)f(x)^{-2}$$

Formulas for exp(x+y), sin(x+y), cos(x+y)

$$\exp(x+y) = e^{x+y} = e^x e^y$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

* Key Limits: limits as $h \to 0$

$$\lim_{h\to 0} (\exp(h) - 1)/h = \lim_{h\to 0} \frac{\exp(0-1) - \exp(0)}{h}$$
$$= 1$$

$$\lim_{h\to 0} \sin(h)/h = \lim_{h\to 0} \frac{\sin(0+h) - \sin 0}{h}$$
$$= \sin'(0) = 1$$

$$\lim_{h\to 0} (\cos(h) - 1)/h = \lim_{h\to 0} \frac{\cos(0+h) - \cos(0)}{h}$$
$$= \cos'(0) = 0$$

Derivatives of Key Functions

x^n	nx^{n-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
e^x	e^x
b^x	$(\log_e b)(b^x)$
$\ln(x)$	x'/x
$\log_b(x)$	$1/(x \log_e b)$

Logarithm Rules

$$\ln(xy) = \ln x + \ln y$$

$$\ln(x/y) = \ln x - \ln y$$

$$\ln(x^n) = n \ln x$$

Trig Rules

x	$\cos(x)$	$\sin(x)$	Degrees
0	1	0	$0°$
$\pi/6$	$\sqrt{3}/2$	$1/2$	$30°$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	$45°$
$\pi/3$	$1/2$	$\sqrt{3}/2$	$60°$
$\pi/2$	0	1	$90°$

$$\sin(x + \pi/2) = \sin x \cos(\pi/2) + \cos x \sin(\pi/2)$$

$$\cos(x + \pi/2) = \cos x \cos(\pi/2) - \sin x \sin(\pi/2)$$