MATH 103: PRACTICE PROBLEMS FOR FIFTH MIDTERM AS WELL AS THE HW DUE AFTER THANKSGIVING BREAK

Do all the problems below for homework. Every problem except the induction at the end is similar to a possible exam question.

Question 1 : Using the definition of the derivative, calculate the derivative of $f(x) = 5x^2 - 2x + 1$. Find the equation of the tangent line at $x = 2$, and use the tangent line to approximate $f(1.97)$.

Question 2 : Calculate the derivative of $f(x) = 3\cos(3x^2 - 12) + \pi\sin(\pi x) - \ln\left(\frac{2}{x}\right)$. Find the equation of the tangent line at $x = 2$, and use the tangent line to approximate $f(2.02)$.

Question 3 : Find the derivatives of the following:

- $A(x) = \ln\left(3x^4 + 2x^\pi - 3x^{7/3} - x^{-1/2}\right)^4 \cos(x)$.
- $B(x) = \frac{(3xe^x - 2\cos(2x) - 1)^5}{2\sin(x)\cos(x)}$.
- $C(x) = \frac{xe^{2x} - x\ln(3x)}{5x}$.
- $D(x) = (\sin(\ln x))^{5/2}$.
- $E(x) = e^{3x^2\ln(x) - 2x} = \exp(3x^2\ln x - 2x)$.

Question 4 : Using implicit differentiation, find $y'(x)$ in terms of $y(x)$ and x, given that $x^2y^3 - \sin(x^2y^2) = \pi^2$. If possible, find how fast y is changing when $x = 1$ and $y = 0$. If possible, find how fast y is changing when $x = \pi$ and $y = 1$.

Question 5 : Using implicit differentiation, find $y'(x)$ in terms of $y(x)$ and x, given that $x^4 + y^5 + \cos(x)\sin(y) = 1$. If possible, find how fast y is changing when $x = 1$ and $y = 0$. If possible, find how fast y is changing when $x = 0$ and $y = 1$.

Question 6 : Find the maximum and minimum values for $f(x) = \frac{1}{3}x^3 - 9x^2 + 80x + 1$ when $-20 \leq x \leq 40$. Use the first and second derivative tests to classify the local maximum and minimums, and sketch the curve.

Question 7 : Find the maximum and minimum values for $f(x) = \sin^2(x) = (\sin x)^2$ when $0 \leq x \leq 2\pi$. Use the first and second derivative tests to classify the local maximum and minimums, and sketch the curve.

Question 8 : Consider all rectangles with perimeter 100. Find the rectangle with largest area.

Question 9 : Consider a sphere whose surface area is growing at 2 meters^2/sec. How fast is the radius increasing when the radius is 2 meters? How fast is the volume increasing when the radius is 3 meters?

Question 10 : State the fundamental theorem of calculus (FTC). (1) Use the FTC to calculate the area under the curve $f(x) = x^2+2x+1$ from $x = 1$ to $x = 4$; (2) use the FTC to calculate the area under the curve of $f(x) = \sin(x)$ from $x = 0$ to $x = \pi/2$. Note we may denote these areas by $\int_1^4 (x^2 + 2x + 1)dx$ and $\int_0^{\pi/2} \sin(x)dx$.

Question 11 : Find *all* the anti-derivatives of the following: (1) x^4; (2) $x^4 + 3x^5$; (3) $(x + 6)^8$; (4) $(x^3 + 4x^2 + 1)^7 \cdot (3x^2 + 8x)$; (5) $\sin(x) - \cos(x) + e^x$.

Question 12 : State L'Hopital's rule. Determine $\lim_{x\to 0} \frac{\sin(x)}{x}$, $\lim_{x\to 0} \frac{\sin(x)\cos(x)-x}{x^2}$, $\lim_{x\to 2} \frac{x^2-4}{(x-2)\sin(x)}$.

Question 13 : Describe mathematical induction (i.e., state what the basis case is, what the inductive step is, and why those two together give a valid proof). Prove

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

Use this to find the area under the curve $f(x) = x^3$ from $x = 0$ to $x = 1$ by computing either the limit of the upper or the lower sum.