The Mathematical Muse: Using Math to Construct Poetry

Patrick Bahls

I fit childish insights within rigid limits, writing shtick which might instill priggish misgivings in critics blind with hindsight.

— Christian Bök, Eunoia

A (Very) Brief History of Mathematical Poetry

You're likely at least somewhat familiar with some of the classical forms of poetry in which basic math plays a passing roll. For example, in crafting tight heroic hexameter, iambic pentameter, and *haiku*, you must keep your eye on the number of syllables you produce in each line. In crafting a sonnet or a sestina, you must count lines (which must obey additional structural rules) rather than syllables. These forms of poetry date back hundreds, if not thousands, of years.

More modern authors have devised poetic constraints of a more intentional, and often more sophisticated, mathematical nature. One of the movements responsible for inventing a substantial number of such constraints is *Oulipo*. Oulipo is a French acronym for ***Ou**vroir de **Lit**térature **Po**tentielle*, a phrase that may be loosely translated as "workshop for potential literature."

Oulipo was founded in November of 1960 by a group of French thinkers led by the writer Raymond Queneau and the mathematician François le Lionnais. Their goal from the outset was to discover constraints that govern the forms which literature can possibly take, and to use those forms to explore literature's full potential. For these writers and those who have followed them, the joy in crafting mathematical poetry lies as much in the creation of new structures for poems as it does in the writing of the poems themselves. Therefore much of their efforts are focused on creating poetic *forms* rather than actual poems: "the goal of the Oulipo is not to give birth to literary works," said Jean Lescure in his "Brief history of the Oulipo."

It's worth noting that to the Oulipians constraint is something desirable: constraint does not confine the writer, rather, it frees him by providing structure within which it's easier to work with the resources language offers him. Put another way, constraint "foregrounds" the structures and forms that underlie the language we use every day, making the artist more aware of those conventions and more able to put them to use in his art.

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Keep in mind that mathematics rests heavily on constraint as well. Imagine, for instance, if you were asked to say all that you could about an arbitrary real-valued function. Without knowing anything else about its structure, there's not much to tell: an arbitrary function has no specific properties and no symmetry of any kind, and it obeys no laws. However, once you impose even a mild constraint, like continuity, you can say a good deal more. Continuous functions can be integrated, they satisfy the Intermediate and Extreme Value Theorems, and they can be approximated arbitrarily closely using polynomials. A small measure of constraint has highlighted useful structure and form, opening up a world of interesting mathematical possibilities.

Poet, novelist, and Oulipo co-founder Raymond Queneau.

In a similar fashion, literary constraint has helped Oulipians to construct a number of delightful poems, novels, and other works of art. To help us get a sense of the sorts of constraint the

There are 10^{14} potential sonnets in this single (sliced up) book.

Oulipians have developed, let's take a quick look at a few of the more explicitly mathematical structures they invented.

Cent Mille Milliards de poèmes. This work of literature, whose title is often translated as "One hundred thousand billion poems," was written by Raymond Queneau in 1961. It consists of ten sonnets (of the required 14 lines apiece) sharing a common rhyme structure so that the reader may select a first line from any of the poems, followed by a second from any of the poems, and then a third, and so forth, and any such selection of lines will yield a grammatically and stylistically "valid" sonnet. Since any line can be selected independently of any other, we easily see that 10^{14} possible combinations are permitted, giving just this many distinct poems. Of course, no one could *actually* write this many poems down, but the structure of the piece contains every one of these poems in its *potential* form.

We should take care not to mistake any single one of the sonnets we construct as a representative for the body of possible selections. From the Oulipian's point of view, the poem is fully realized only in our minds when we consider the entire body of potential poems as a whole.

The $S+7$ Algorithm. Oulipo co-founder Raymond Queneau credits his colleague Jean Lescure with inventing this "perimathematical" means, as he called it, of creating a new literary work from an old one. Given an existing text, we modify the text by replacing every noun with the 7th noun following it in a predetermined dictionary. The result is a sort of mad-lib in which all of the nouns in the original work have been replaced.

Of course, the choice of the number 7 is entirely arbitrary, and you can replace adjectives, verbs, or other parts of speech instead of nouns. Moreover, you need not even rely on a dictionary to govern your replacements. Here, for example, is the result of rewriting one of Emily Dickinson's poems using Mary Shelley's *Frankenstein* as the "replacement" text:

> The pedigree of honey
> Does not concern the bee;
> A clover, any time, to him
> is aristocracy.

becomes

> The pole of horizon
> Does not concern the breeze;
> A commencement, any time, to him
> is agitation.

Although the poems arising from this procedure may seem random, we must remember that their construction is governed by a very specific rule, and they are therefore not random at all. Indeed, the Oulipians are careful to point out that their works are not random, and they firmly resist attempts by others to label their writing as products of chance.

Lipograms. A lipogram is a work of art from which a certain letter or set of letters are intentionally omitted from all or a part of the work. For instance, Oulipian Georges Perec's 1969 novel *La disparition* excludes the letter 'E' entirely, as does Ernest Vincent Wright's 1939 novel *Gadsby*. Meanwhile, Christian Bök's 2001 book *Eunoia* comprises five chapters, each of which excludes all but a single vowel. The epigram at the start of this article shows off a line from Chapter I, a line which, fittingly, elevates literary constraint. Here's another taste, the first few sentences from Chapter A:

> Awkward grammar appals a craftsman. A Dada bard as daft as Tzara damns stagnant art and scrawls an alpha (a slapdash arc and a backward zag) that mars all stanzas and jams all ballads (what a scandal). A madcap vandal crafts a small black ankh—a hand-stamp that can stamp a wax pad and at last plant a mark that sparks an *ars magna* (an abstract art that charts a phrasal anagram).

Another recent lipogrammatic work of note is Mark Dunn's 2001 novel *Ella Minnow Pea*, a book in which letters are dropped from usage one at a time as the novel progresses.

Oulipo is alive and well, and since 1960 dozens of artists and thinkers have counted themselves among its members. If you're interested in reading more about Oulipo, Warren Motte's *Oulipo: a Primer of Potential Literature*, 2nd ed. (Champaign: Dalkey Archive Press, 2007) is an excellent resource, and various websites have brought their poetic works closer than ever before to their full potential.

New Means of Poetic Invention

It's time for us to develop a brand new mathematical form of poetic constraint, one very much in the spirit of Oulipo. In fact, as we'll soon see, our "new" form gives a nice way of describing both the $S+7$ algorithm and lipograms precisely.

As you learn in an undergraduate abstract algebra course, a *group* is a set G equipped with an associative binary operation $\circ$ such that there is an identity, ϵ, with respect to $\circ$ and every element g in G has an *inverse*, g^{-1}, satisfying $g \circ g^{-1} = g^{-1} \circ g = \epsilon$. If you haven't seen groups before, it might help you to think about the set of integers, $Z = \{\ldots,-2,1,0,1,2,\ldots\}$, with the operation of addition, +. In the group Z, $\epsilon = 0$ since $n+0 = 0+n = n$ for all integers n, and for a given integer n, the number $-n$ gives us an inverse, since $-n+n = n+(-n) = 0$.

Here's another way to define a group, called a *free group*, which we'll denote by F. The group F is comprised by all possible strings of letters from the English alphabet, whether those strings form valid words or not. The operation in this group is simply *concatenation*: to "multiply" two words together, simply place them end-to-end. The *empty word* ϵ, consisting of no letters at all, serves as our identity. (Though we won't need them, it's possible to define inverses as well.)

One of the nice things about the free group F is that it's very easy to define functions from F to another group G, functions which preserve the structure of the group in a very natural way. If G is any group and we define the value of some function f on each of the single letters in F, we can define f on all of F just by multiplying together the images of the single letters. For example, if we know the values of $f(A)$, $f(B)$, and so forth, then it makes sense to define

$$f(HELLO) = f(H) \circ f(E) \circ f(L) \circ f(L) \circ f(O)$$

when $\circ$ is the operation in G. A function like this one, one that preserves the multiplication on the group F, is called a *group homomorphism*.

We're now just about ready to write some poems. The only missing ingredient is some sort of *constraint* to guide our creation, and this can easily be supplied by the right choice of group homomorphism on F.

For instance, let's consider the very simple homomorphism f from F to Z defined by

$$f(x) = \begin{cases} 1 \text{ if } x \text{ is a vowel; and} \\ -1 \text{ if } x \text{ is a consonant.} \end{cases}$$

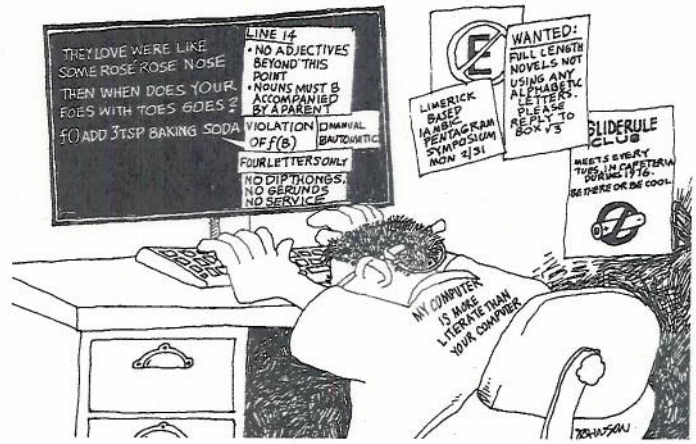
We then require that whatever poem we write have the property that when we apply f to the poem in its entirety, the value is 0. I've written the following simple poem to meet this criterion (whether or not you include its title):

A balance

O, Beauty, abound! O, Beauty, prevail!
O, Consonant, Vowel, be laid on the scale!

Of course, all we're really asking here is that the poem contain the same number of vowels as consonants. The poem is rather contrived, but it would be much easier to construct "balanced" poems in languages (like Spanish, Italian, or any other romance language) in which the ratio of vowels to consonants is higher than it is in English.

A slightly more complicated example of a homomorphism constraint requires the group we'll denote Z_{26}. This group of integers *modulo* 26 consists of the numbers {0, 1, ..., 24, 25}, on which we may define a sort of "wraparound" addition, $\oplus$. For $m, n \in Z_{26}$, $m \oplus n$ is defined in the ordinary fashion (that is, $m \oplus n = m + n$) if $m + n \leq 25$, but if $m + n$ exceeds 25, we must wrap around by starting over again at 0, so that $m \oplus n = m + n - 26$. You can also compute $m \oplus n$ by finding the



Cartoon by John Johnson

remainder of $m + n$ upon division by 26. This observation makes it easy to compute sums

$$m_1 \oplus m_2 \oplus \cdots \oplus m_k \text{ in } Z_{26}$$

since all you need to do is find the remainder of $m_1 + m_2 + \cdots + m_k$ upon division by 26.

How can we use this group to guide us in our poetic adventures? Let's define the homomorphism from F to Z_{26} by $f(A) = 1, f(B) = 2, \ldots, f(Y) = 25, f(Z) = 0$.

Then we require that our poem have the property that $f(\ell_1) = f(\ell_2)$ for any two lines ℓ_1 and ℓ_2 in the poem. That is, every line has the same value under the homomorphism f.

Here's a poem I've penned with this property, in which $f(\ell) = 6$ for every line ℓ, as you should check:

Six

Are we so free that we must build
tight cages out of self-wrought bars?,
that, every barrier of old o'erperched,
bold walls we must before us raise to
repress our over-anxious powers?
Dreary now is Spenser's sonnetry,
and empty are Keats's odes:
a crowd of words must measure more
than the essence it encodes.

So vowing, to this priesthood
I, in metered ciphered characters,
make an offer of my novice oath,
though a cheap and clumsy canticle,
in a passionately davened prayer.

There's actually a bit more structure than we bargained for here: because the poem has 14 lines and every line gives the value 6, applying f to the whole poem P gives the remainder of $14 \cdot 6 = 64$ upon division by 26. But this is 6 again! Moreover, you can check that $f(SIX) = 0$, so that we can even throw the title in and it doesn't change the fact that the poem (with title) gives the value 6.

Group homomorphisms give us a tremendously flexible tool for developing new poetic constraints. In fact, one of the Oulipian methods we saw earlier can be recaptured easily by the right choice of group homomorphism. If our goal is to construct a lipogram that avoids using any of the letters in a certain set S, we can define the homomorphism f from F to Z by $f(x) = 1$ if x is in S and $f(x) = 0$ if x is not in S. It's not hard to see that a poem P is now a lipogram if and only if $f(P) = 0$.

Where do we go from here?

It's clear that we've only surveyed the tip of the iceberg here: there are countless groups G with which we could work, and for each group G there are generally many homomorphisms f from F to G. There are particularly clever choices of G and f yet to be uncovered.

For instance, might we choose G and f that exploit the shape of the letters themselves, incorporating the letters' symmetry? Or perhaps we're not happy defining f only on letters and we wish to extend it to operate on punctuation, spaces, and line breaks as well.

Moreover, there are ways in which we can incorporate even richer mathematical structure into our construction. For instance, what if instead of considering groups and group homomorphisms we consider *rings* and *ring homomorphisms*? Perhaps we might insist that a space between words (or a line break, or a break between stanzas) signals that we perform one operation in the ring and that ordinary juxtaposition of characters signals that we perform the other.

The course we've taken in the past few pages has traced a continuous path from some of the "classical" poetic constraints developed decades ago to brand new ones involving some advanced mathematics. My intent has been not only to showcase some of the forms of art various authors have developed using literary constraints, but also to shine a light on the common ground shared by mathematics and literature, emphasizing the crucial role of Oulipo in exploring that ground. We ought to keep in mind that the poet and the mathematician work towards many of the same goals: both seek to communicate to others some sort of truth about the world in which we live, and both seek to discover and describe structure where there was no such structure before.

I hope our brief journey has proven both intriguing and inspiring. Indeed, I hope you now feel motivated to write a few poems of your own, either based upon the constraints described above or upon entirely new ones of your own devising. Mathematics offers an incomprehensible wealth of tools and techniques with which to work, so there's a great deal of room left to explore. Though the world-building work of a poet is often arduous, it's also tremendously fun, and I hope that you can find delight in discovery as you find freedom in constraint. ■

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