MATH 209: HW DUE APRIL 22ND

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Question 1 (**40 points**) : Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix}.$$

Show the eigenvectors are $\binom{1}{2}$ and $\binom{1}{3}$. Find the matrix S that diagonalizes A, and solve the first order linear non-homogenous system of differential equations

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = A \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \exp(-t) \\ \exp(3t) \end{pmatrix}.$$

Question 2 (**40 points**) : Find all solutions to the following (or prove no solution exists):

(1) $y'(x) + xy(x) = \exp(-x^2/2)$ with $y(0) = 1$.
(2) $(x^2 + 4x + 4)dy/dx = \exp(-y)$.
(3) $y''(x) + (x^2 + 1)y'(x) + y(x) = 0$.
(4) $y''(x)+2y'(x)-15y(x) = x^2+x+1$ with $y(0) = 1$ and $y'(0) = 1$.

Question 3 (**20 points** : **extra credit**) : Let F_n be the nth Fibonacci number ($F_{n+2} = F_{n+1} + F_n$ with $F_0 = 0$ and $F_1 = 1$). Evaluate $\sum_{n=0}^{\infty} F_n/3^n$ (or show the sum does not converge).

Date: April 17, 2009.