MATH 209: HW DUE APRIL 29ND

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Question 1 (**30 points**) : Using the Euler (or tangent line) method with step size $h = .5$ and $h = .25$, find the approximate values of the solution to $y'(t) = 3 + t - y(t)$ with $y(0) = 1$ at the points 0, .25, .5, .75 and 1. Compare your answer to the exact solution to this equation.

Question 2 (**20 points**) : Let

$$A \ = \ \begin{pmatrix} 1 & 2 & \pi \\ \sqrt{7} & 5 & 0 \\ 2 & e & 1 \end{pmatrix}, \quad \overrightarrow{x}(0) \ = \ \begin{pmatrix} 1 \\ \pi \\ \sqrt{2009} \end{pmatrix}.$$

Find the solution to $\overrightarrow{x}'(t) = A\overrightarrow{x}(t)$ with A and $\overrightarrow{x}(0)$ as above. *Hint: you don't need to be able to work with your solution, just write the solution down.*

Question 3 (**20 points**) : (1) Solve $y'(t) = ay(t) + by(t)^2$ with parameters $a, b > 0$ and $y(0) = y_0 \geq 0$. (2) Solve $y''(t) + y(t) = \tan(t)$ for $0 < t < \pi/2$. *Hint: some of the techniques from chapter 3 might be useful.*

Question 4 (**10 points**) : Let $f(x)$ be a continuous, strictly increasing differentiable function from $[0, \infty)$ to the real numbers such that

$$(f(x))^3 \ = \ \int_0^x t\,(f(t))^2\,dt$$

for any $x \geq 0$. Determine $f(x)$ for all $x \geq 0$. *Aside: this problem* is *similar to something we have done previously in class, and thus there is a reason it is included in the homework!*

Date: April 27, 2009.