```
(* loads in packages we need *)
```

In[659]:=
```
Needs["MultivariateStatistics`"]
Needs["Histograms`"]
Needs["BarCharts`"]
```

In[662]:=
```
(* this generates random numbers
 from the normal distribution *)
Random[NormalDistribution[0, 1]]
?NormalDistribution
```

Out[662]= 0.00317267

NormalDistribution[μ, σ] represents a normal (Gaussian) distribution with mean μ and standard deviation σ.

NormalDistribution[] represents a normal distribution with zero mean and unit standard deviation. ≫

In[786]:=
```
(* this is the main program. the first
 part generates a random matrix A where
 it is one of three possible matrices;
the second generates a random matrix
 B where the various key entries are
 taken from normal distributions;
Bexact is the same as B but with 0 variance. *)
f[x_] := If[x < .1, 1 / 6, If[x < .9, 2 / 6, 3 / 6]];
A[x_] := {{0, f[x], 2.005 / 3, 0},
   {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}};
B[x_] := {{0, Abs[Random[
      NormalDistribution[2 / 5.11, .06]]],
    Abs[Random[NormalDistribution[x 2.005 / 3, .1]]],
    0}, {.95 + Random[] * .05, 0, 0, 0},
   {0, .9 + Random[] * .1, 0, 0},
   {0, 0, .8 + Random[] * .2, 0}};
Bexact = {{0, 2 / 5.11, 2.005 / 3, 0},
   {.975, 0, 0, 0}, {0, .95, 0, 0}, {0, 0, .9, 0}};
Bexactvar[x_] := {{0, 2 / 5.11, x 2.005 / 3, 0},
   {.975, 0, 0, 0}, {0, .95, 0, 0}, {0, 0, .9, 0}};
MatrixForm[B[1]]
MatrixForm[Bexact]
Eigenvalues[Bexact]
Eigenvalues[Bexactvar[1.1]]
Eigenvalues[Bexactvar[1.2]]
Eigenvalues[Bexactvar[1.3]]
(*A[.08]
 A[.8]
 A[.93]*)
```

SetDelayed::write :

Tag List in {{0, 2, −1}, {−1, 0, 2}, {1, −1, 0}}[x_] is Protected. ≫

Out[791]//MatrixForm=

$$\begin{pmatrix} 0 & 0.438811 & 0.719657 & 0 \\ 0.973081 & 0 & 0 & 0 \\ 0 & 0.999673 & 0 & 0 \\ 0 & 0 & 0.801641 & 0 \end{pmatrix}$$

Out[792]//MatrixForm=

$$\begin{pmatrix} 0 & 0.391389 & 0.668333 & 0 \\ 0.975 & 0 & 0 & 0 \\ 0 & 0.95 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \end{pmatrix}$$

Out[793]= {1.00025, −0.500124 + 0.607262 i, −0.500124 − 0.607262 i, 0.}

Out[794]= {1.02326, −0.511632 + 0.635373 i, −0.511632 − 0.635373 i, 0.}

Out[795]= {1.04516, −0.52258 + 0.661562 i, −0.52258 − 0.661562 i, 0.}

Out[796]= {1.06606, −0.533032 + 0.686122 i, −0.533032 − 0.686122 i, 0.}

In[671]:=
```
(* testing how to take entries of a vector *)
v0 = {{1}, {0}, {0}, {0}};
v0[[1, 1]] + v0[[2, 1]]
```

Out[672]= 1

In[813]:=
```
(* main program to look at evolution *)
(* if doB = 1 then we use the random B matrices,
else use the A matrices *)
(* saveevery tells how many steps
 we take before saving to file *)
(* growth = 1 is the test case where the largest
  eigenvalue is 1.00025, just slightly above 1;
if we use larger growth values then we can have
 a better chance of the population surviving. *)
evolve[years_, iterations_, doB_,
   saveevery_, growth_] := Module[{},
   allresults = {};
   allratios = {};
   exact = {};
   (* initial population *)
   v0 = {{1}, {0}, {0}, {0}};
   initialpop =
    v0[[1, 1]] + v0[[2, 1]] + v0[[3, 1]] + v0[[4, 1]];
   exact = AppendTo[exact, {0, initialpop}];
   Print[
```

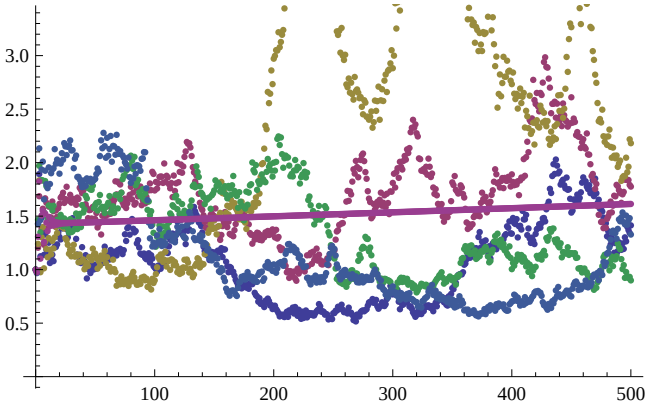
```
 "Largest eigenvalue of the constant matrix is ",
 N[Eigenvalues[Bexactvar[growth]][[1]]];
For[i = 1, i ≤ iterations, i++,
 {
  (* saves in exact, results and ratios *)
  results = {{0, initialpop}};
  ratio = {{0, 1}};
  vcurr = v0;
  vexactcurr = v0;
  (*Print[vcurr];*)
  For[n = 1, n ≤ years, n++,
   {
    If[doB == 1, vcurr = B[growth].vcurr,
     vcurr = A[Random[]] . vcurr];
    If[Mod[n, saveevery] == 0,
     results = AppendTo[results,
       {n, vcurr[[1, 1]] + vcurr[[2, 1]] +
         vcurr[[3, 1]] + vcurr[[4, 1]]}]];
    If[i == 1,
     {
      If[doB == 1,
       vexactcurr = Bexactvar[growth].vexactcurr,
       vexactcurr = A[.5] . vexactcurr];
      If[Mod[n, saveevery] == 0,
       exact = AppendTo[exact, {n, vexactcurr[[1,
```

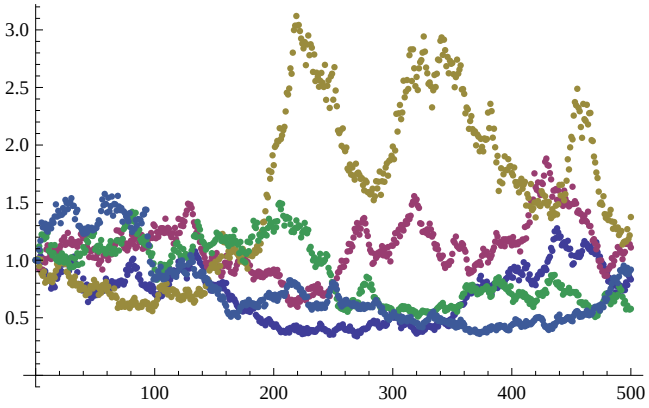
```
                1]] + vexactcurr[[2, 1]] + vexactcurr[[
                3, 1]] + vexactcurr[[4, 1]]}]];
          }];
         If[Mod[n, saveevery] == 0,
          ratio = AppendTo[ratio,
            {n, results[[n / saveevery, 2]] /
               exact[[n / saveevery, 2]] 1.0}]];
        }]; (* end of n years loop *)
       allresults = AppendTo[allresults, results];
       allratios = AppendTo[allratios, ratio];
       Print["At time ",
        n, " we have a population of ",
        results[[Length[results], 2]],
        " giving a ratio to the current exact
          population of ",
        ratio[[Length[ratio], 2]]];
      }]; (* end i iterations loop *)
     allresults = AppendTo[allresults, exact];
     Print[ListPlot[allresults]];
     Print[ListPlot[allratios]];
    ];
In[833]:= evolve[500, 5, 1, 1, 1]
    evolve[500, 5, 1, 1, 1.001]
    evolve[500, 5, 1, 1, 1.01]
Largest eigenvalue of the constant matrix is 1.00025
At time 501 we have a population of 1.41354
  giving a ratio to the current exact population of
 0.832617
At time 501 we have a population of 1.77595
  giving a ratio to the current exact population of
 1.11041
```

At time 501 we have a population of 2.18135
giving a ratio to the current exact population of 1.37479

At time 501 we have a population of 0.899479
giving a ratio to the current exact population of 0.577833

At time 501 we have a population of 1.32787
giving a ratio to the current exact population of 0.917932



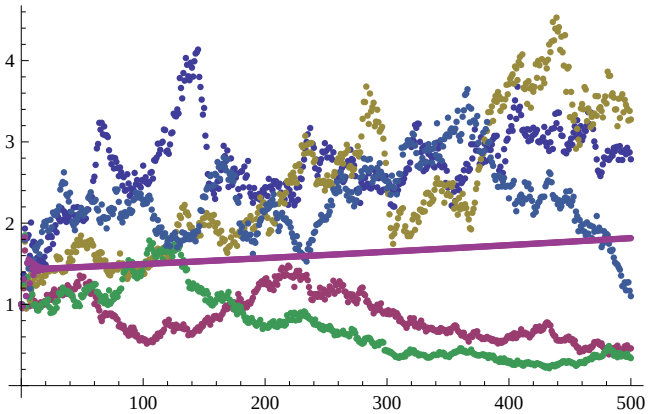


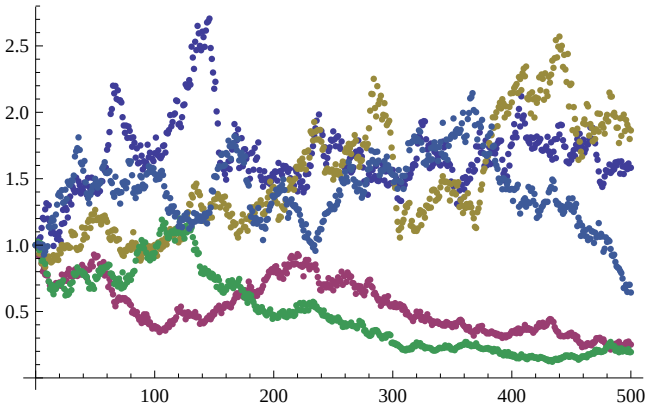
Largest eigenvalue of the constant matrix is 1.00048

At time 501 we have a population of 2.78597
giving a ratio to the current exact population of 1.58283

At time 501 we have a population of 0.458955
giving a ratio to the current exact population of 0.251292

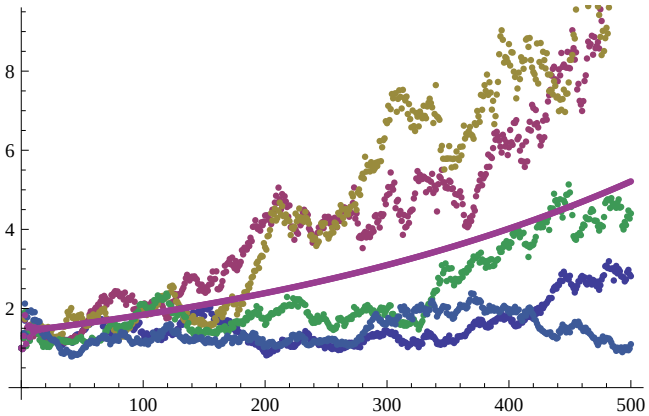
```
At time 501 we have a population of 3.27572
  giving a ratio to the current exact population of
 1.86363
At time 501 we have a population of 0.338633
  giving a ratio to the current exact population of
 0.194733
At time 501 we have a population of 1.10138
  giving a ratio to the current exact population of
 0.643344
```

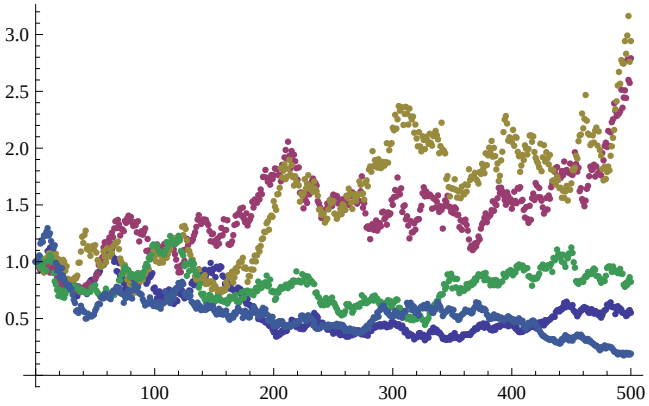





```
Largest eigenvalue of the constant matrix is 1.0026
At time 501 we have a population of 2.81678
  giving a ratio to the current exact population of
 0.550506
At time 501 we have a population of 13.7303
  giving a ratio to the current exact population of
 2.79092
```

```
At time 501 we have a population of 14.5783
  giving a ratio to the current exact population of
 2.94247
At time 501 we have a population of 4.40286
  giving a ratio to the current exact population of
 0.822122
At time 501 we have a population of 1.0975
  giving a ratio to the current exact population of
 0.190139
```

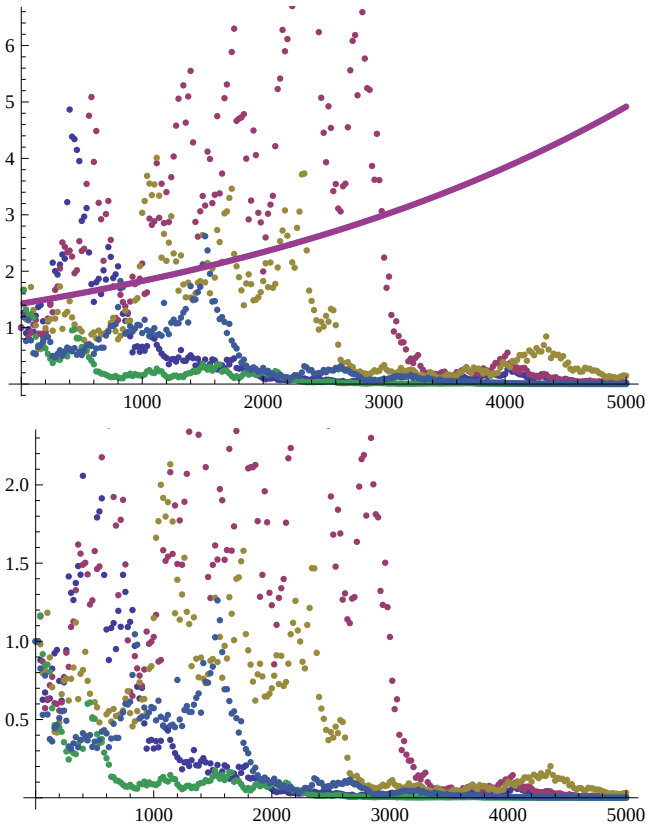




In[830]:=
```
evolve[5000, 5, 1, 20, 1]
evolve[5000, 5, 1, 20, 1.001]
evolve[5000, 5, 1, 20, 1.01]
```

```
Largest eigenvalue of the constant matrix is 1.00025
```

```
At time 5001 we have a population of 0.0261422
  giving a ratio to the current exact population of
 0.00625655
At time 5001 we have a population of 0.0172636
  giving a ratio to the current exact population of
 0.00375957
At time 5001 we have a population of 0.146734
  giving a ratio to the current exact population of
 0.0316139
At time 5001 we have a population of 0.0000820645
  giving a ratio to the current exact population of
 0.0000211209
At time 5001 we have a population of 0.00330274
  giving a ratio to the current exact population of
 0.000546533
```




```
Largest eigenvalue of the constant matrix is 1.00048
```

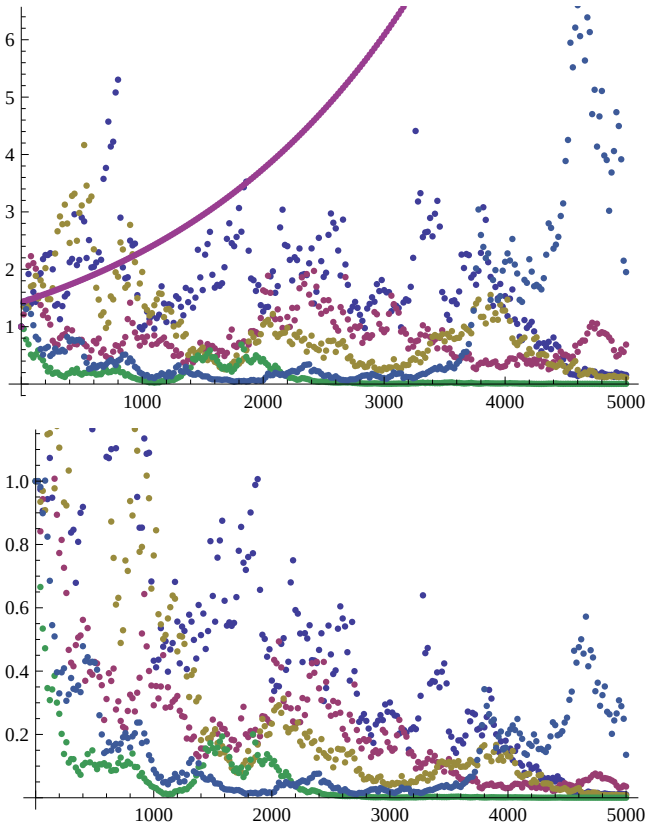
At time 5001 we have a population of 0.159159
  giving a ratio to the current exact population of
 0.0104628

At time 5001 we have a population of 0.688041
  giving a ratio to the current exact population of
 0.0353092

At time 5001 we have a population of 0.110283
  giving a ratio to the current exact population of
 0.00782502

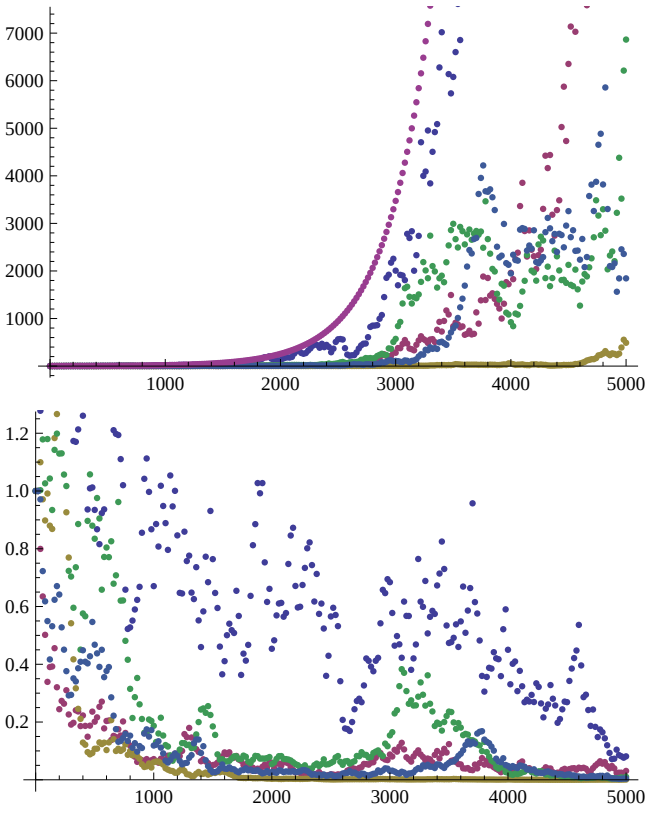
At time 5001 we have a population of 0.00390076
  giving a ratio to the current exact population of
 0.000233547

At time 5001 we have a population of 1.95082
  giving a ratio to the current exact population of
 0.135617



Largest eigenvalue of the constant matrix is 1.0026

```
At time 5001 we have a population of 58105.4
  giving a ratio to the current exact population of
 0.0811533
At time 5001 we have a population of 15271.8
  giving a ratio to the current exact population of
 0.0306358
At time 5001 we have a population of 490.392
  giving a ratio to the current exact population of
 0.000925266
At time 5001 we have a population of 6864.5
  giving a ratio to the current exact population of
 0.0103799
At time 5001 we have a population of 1844.29
  giving a ratio to the current exact population of
 0.00393978
```



In[814]:=
```
evolve[50 000, 5, 1, 200, 1]
evolve[50 000, 5, 1, 200, 1.001]
evolve[50 000, 5, 1, 200, 1.01]
```

Largest eigenvalue of the constant matrix is 1.00025

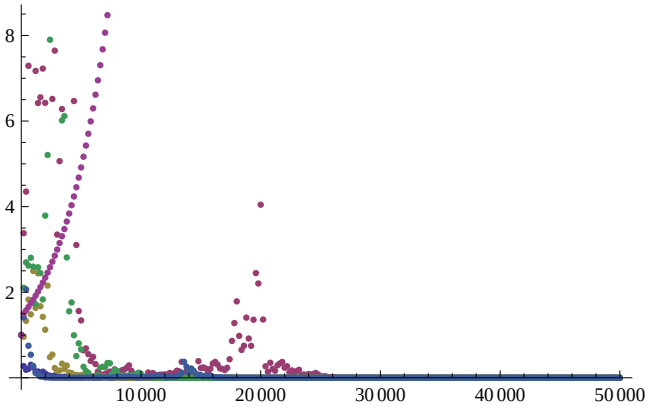
At time 50 001 we have a population of 2.08979×10^{-11}
giving a ratio to the current exact population of
2.13842×10^{-16}

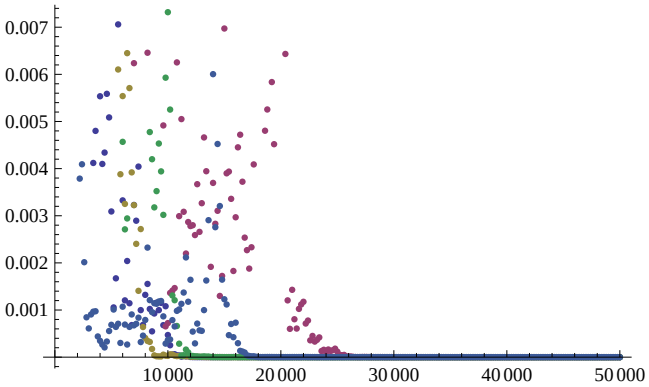
At time 50 001 we have a population of 1.91589×10^{-7}
giving a ratio to the current exact population of
4.91374×10^{-13}

At time 50 001 we have a population of 1.94422×10^{-16}
giving a ratio to the current exact population of
1.93882×10^{-21}

At time 50 001 we have a population of 1.33926×10^{-11}
giving a ratio to the current exact population of
3.834×10^{-17}

At time 50 001 we have a population of 2.32932×10^{-9}
giving a ratio to the current exact population of
8.14522×10^{-15}





Largest eigenvalue of the constant matrix is 1.00048

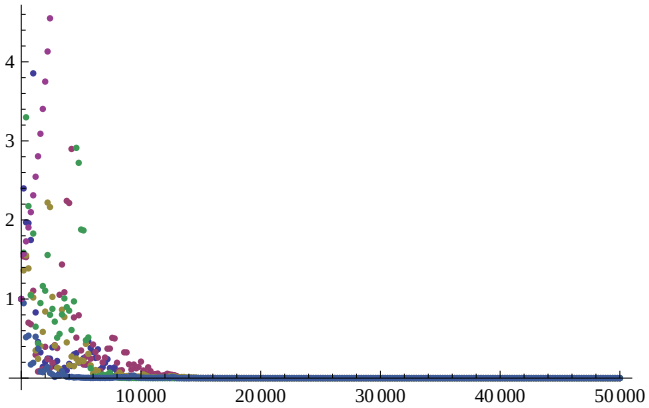
At time 50 001 we have a population of 5.1677×10^{-6}
giving a ratio to the current exact population of
9.50332×10^{-17}

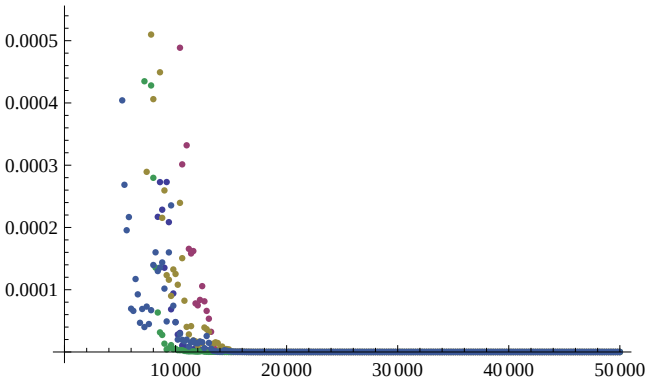
At time 50 001 we have a population of 6.99024×10^{-13}
giving a ratio to the current exact population of
1.44636×10^{-23}

At time 50 001 we have a population of 2.55295×10^{-8}
giving a ratio to the current exact population of
9.1124×10^{-19}

At time 50 001 we have a population of 8.65278×10^{-16}
giving a ratio to the current exact population of
1.78179×10^{-26}

At time 50 001 we have a population of 3.64078×10^{-15}
giving a ratio to the current exact population of
1.51683×10^{-25}





Largest eigenvalue of the constant matrix is 1.0026

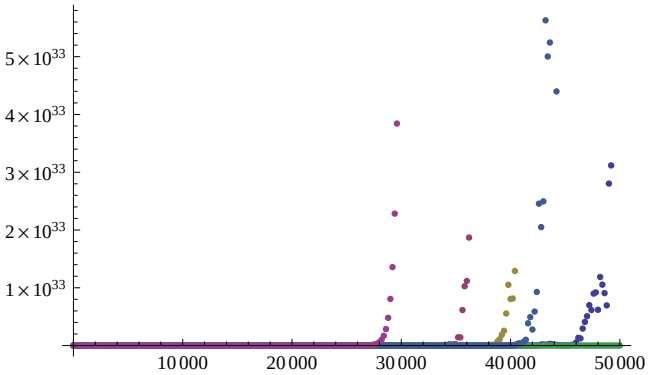
At time 50 001 we have a population of 8.47934×10^{33}
giving a ratio to the current exact population of
3.90712×10^{-23}

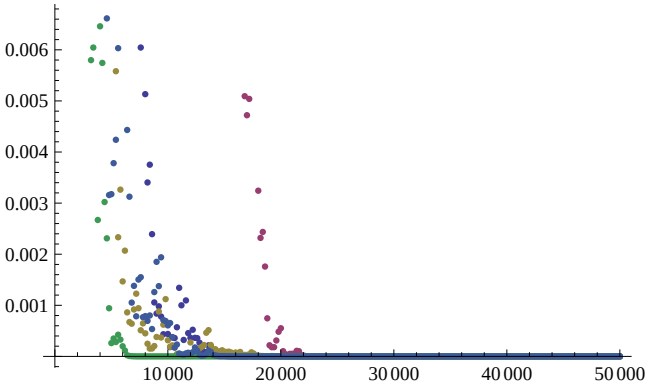
At time 50 001 we have a population of 1.33622×10^{41}
giving a ratio to the current exact population of
2.98391×10^{-16}

At time 50 001 we have a population of 1.97846×10^{40}
giving a ratio to the current exact population of
1.21851×10^{-16}

At time 50 001 we have a population of 6.88594×10^{26}
giving a ratio to the current exact population of
1.76042×10^{-30}

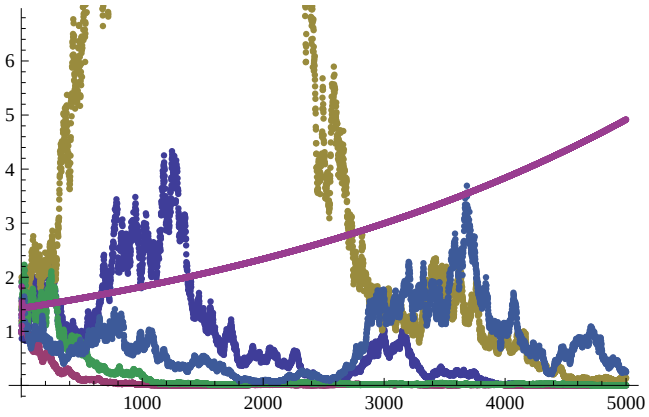
At time 50 001 we have a population of 5.86688×10^{39}
giving a ratio to the current exact population of
3.19551×10^{-17}

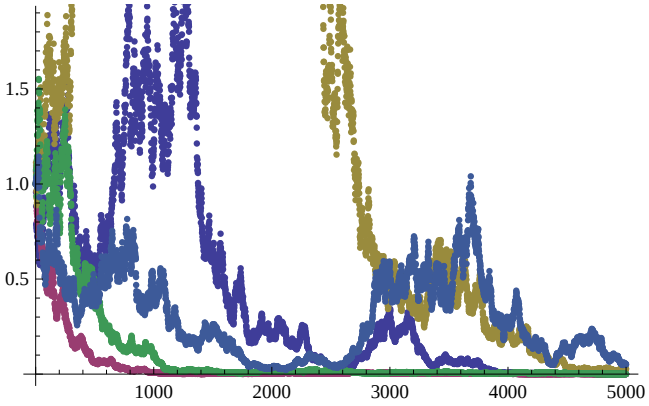




In[822]:= **evolve[5000, 5, 1, 1, 1]**

```
Largest eigenvalue of the constant matrix is 1.00025
At time 5001 we have a population of 0.00305858
  giving a ratio to the current exact population of
 0.000708953
At time 5001 we have a population of 0.00158931
  giving a ratio to the current exact population of
 0.00031694
At time 5001 we have a population of 0.139462
  giving a ratio to the current exact population of
 0.0272917
At time 5001 we have a population of 0.00243459
  giving a ratio to the current exact population of
 0.000513014
At time 5001 we have a population of 0.247656
  giving a ratio to the current exact population of
 0.0514645
```





In[823]:=
```
evolve[5000, 5, 1, 1, 1.3]
```

Largest eigenvalue of the constant matrix is 1.06606

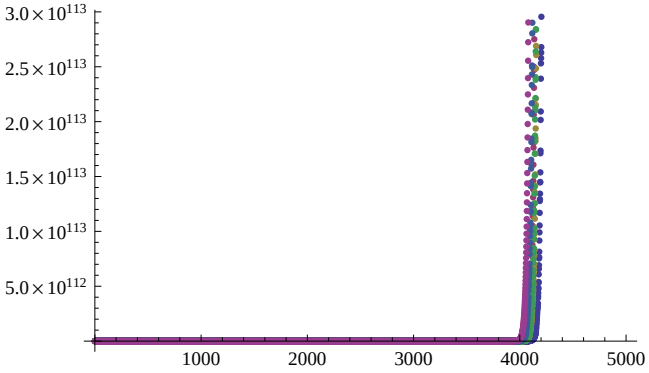
At time 5001 we have a population of 1.4923×10^{136}
giving a ratio to the current exact population of
0.00143177

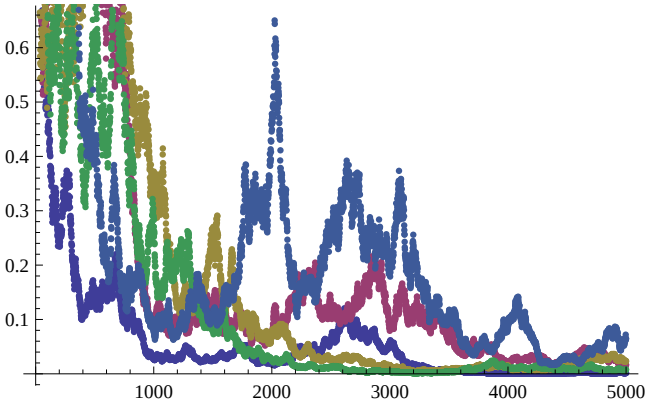
At time 5001 we have a population of 1.79864×10^{137}
giving a ratio to the current exact population of
0.0167185

At time 5001 we have a population of 2.29037×10^{137}
giving a ratio to the current exact population of
0.0230612

At time 5001 we have a population of 8.30768×10^{136}
giving a ratio to the current exact population of
0.0078477

At time 5001 we have a population of 6.6036×10^{137}
giving a ratio to the current exact population of
0.0646244



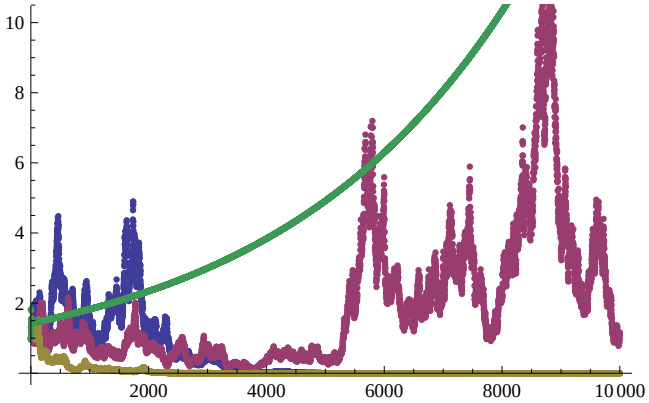


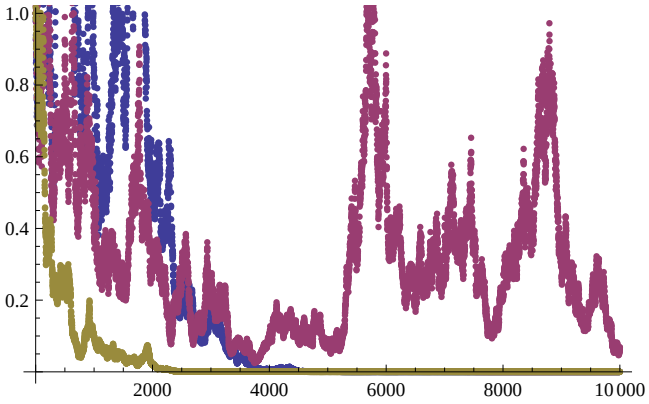
```
evolve[10 000, 3, 1, 1, 1]
```

At time 10 001 we have a population of 3.25221×10^{-6}
giving a ratio to the current exact population of
1.82356×10^{-7}

At time 10 001 we have a population of 1.11556
giving a ratio to the current exact population of
0.0676171

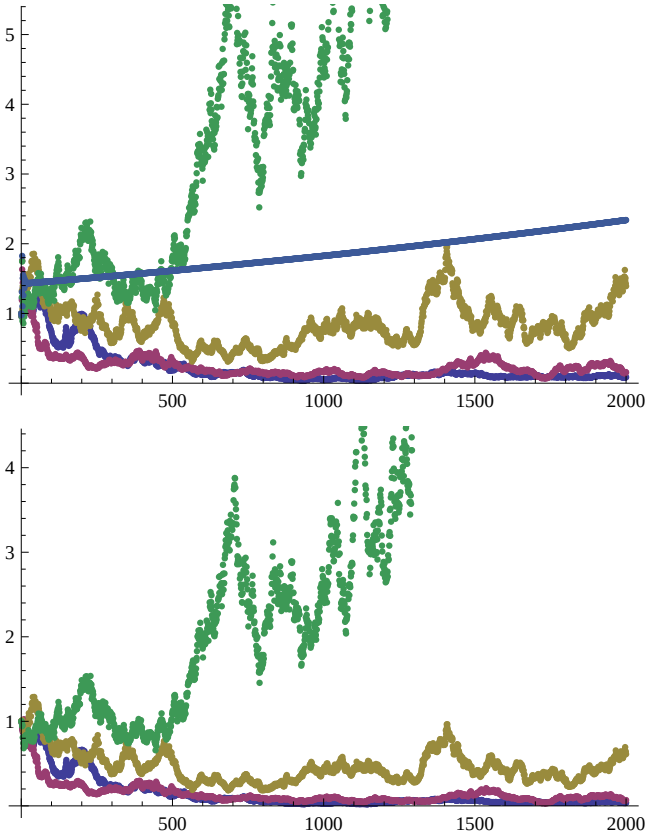
At time 10 001 we have a population of 2.38721×10^{-6}
giving a ratio to the current exact population of
1.30222×10^{-7}





In[824]:= **evolve[2000, 4, 1, 1, 1]**

```
Largest eigenvalue of the constant matrix is 1.00025
At time 2001 we have a population of 0.0854212
  giving a ratio to the current exact population of
 0.0374684
At time 2001 we have a population of 0.15409
  giving a ratio to the current exact population of
 0.0682715
At time 2001 we have a population of 1.3914
  giving a ratio to the current exact population of
 0.611692
At time 2001 we have a population of 76.1452
  giving a ratio to the current exact population of
 33.0045
```



In[828]:=
```
evolve[5000, 4, 1, 20, 1]
evolve[5000, 4, 1, 20, 1.01]
```

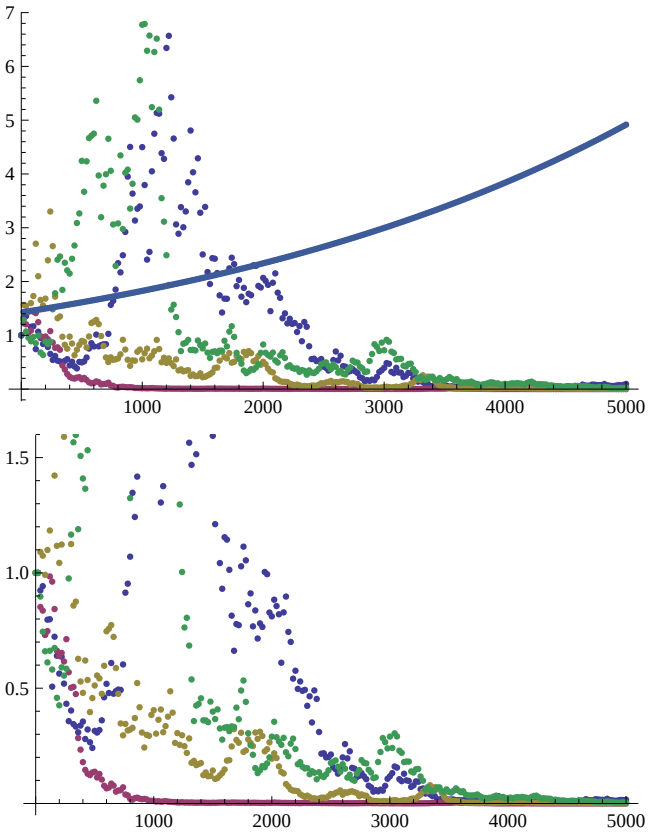
Largest eigenvalue of the constant matrix is 1.00025

At time 5001 we have a population of 0.0950236
giving a ratio to the current exact population of 0.0163902

At time 5001 we have a population of 0.0000438789
giving a ratio to the current exact population of 0.0000125465

At time 5001 we have a population of 0.00571591
giving a ratio to the current exact population of 0.00108537

At time 5001 we have a population of 0.0294447
giving a ratio to the current exact population of 0.00437125



Largest eigenvalue of the constant matrix is 1.0026

```
At time 5001 we have a population of 32 924.2
  giving a ratio to the current exact population of
 0.0555459
At time 5001 we have a population of 100.864
  giving a ratio to the current exact population of
 0.000128061
At time 5001 we have a population of 4898.48
  giving a ratio to the current exact population of
 0.00744516
At time 5001 we have a population of 52 032.9
  giving a ratio to the current exact population of
 0.111651
```