SUMMARY OF *ON THE NUMBER OF PRIME NUMBERS LESS THAN A GIVEN QUANTITY*

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The goal of Riemann's paper *On the Number of Prime Numbers less than a Given Quantity* is to investigate "the accumulation of the prime numbers" quoting the introduction.

The paper contains three parts. First, he defines the zeta function $\zeta(s)$ to be the function of the complex variable s with two expressions

$$\Pi \frac{1}{1-\frac{1}{p^s}} = \sum \frac{1}{n^s}$$

whenever they converge. Using the zeta function and other methods, he defines $\psi(t)$ as

$$\xi(t) = \frac{1}{2} - (tt + \frac{1}{4}) \int_1^\infty \psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2} t \log(x)) dx$$

Second, he analyzes the properties of $\xi(t)$. Third, he lets $F(x)$ be the number of primes less than x is x is not a prime, greater by $\frac{1}{2}$ is x is a prime. Using methods he developed above, he obtains

$$F(x) = \sum (-1)^\mu \frac{1}{m} f(x^{\frac{1}{m}})$$

in which m is the series consisting of natural numbers that are not divisible by any square other than 1, and μ denotes the number of prime factors of m.

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