Prospect Theory

Developed by Daniel Kahneman and Amos Tversky in the paper *Prospect Theory: An Analysis of Decision under Risk* (Kahneman and Tversky, 1979), the prospect theory is a psychologically realistic alternative to the expected utility theory. It describe decision making between alternatives involving risk.

The original theory was motivated by the *Allais Paradox* with the following set up:

Gamble A or Gamble B?

- Gamble A: A 100% chance of receiving \$1 million.
- Gamble B: A 10% chance of receiving \$5 million, an 89% chance of receiving \$1 million, and a 1% chance of receiving nothing.

Gamble C or Gamble D?

- Gamble C: An 11% chance of receiving \$1 million, and an 89% chance of receiving nothing.
- Gamble D: A 10% chance of receiving \$5 million, and a 90% chance of receiving nothing.

The authors observed that between A and B, most people chose B. So using the expected utility theory,

$$E[u(A)] = u(1), \;\; E[u(B)] = 0.1u(5) + 0.89u(1) + 0.01u(0)$$
$$E[u(A)] > E[u(B)] \;\; \Rightarrow \;\; u(1) > 0.1u(5) + 0.89u(1) + 0.01u(0)$$
$$\text{or} \;\; 0.11u(1) > 0.1u(5) + 0.01u(0)$$

Adding $0.89u(0)$ to each side, we get:

$$0.11u(1) + 0.89u(0) > 0.1u(5) + 0.90u(0)$$
$$\text{or} \;\; E[u(C)] > E[u(D)]$$
$$\text{Note: } E[C] = \$0.11\text{million} < \$0.5\text{million} = E[D]$$

Which implies that the same people that chose B over A should also choose C over D. But this is oftentimes not the reality. The two authors thus proposed that when probabilities are low, most people choose the prospect that offers the larger gain, i.e. Gamble D over Gamble C.

The prospect theory characterizes the decision-making process into two stages

(1) Editing: possible outcomes of the decision are ordered following some heuristic.
(2) Evaluation: evaluate the prospects and choose the highest value

The prospect theory differs from the conventional expected utility theory in two respects

- The expected utility says that the final wealth level matters matters, whereas the prospect theory posits that people normally perceive outcomes as gains and losses relative to some reference point, which could be current wealth level, worst scenario outcome, etc.
- The expected utility theory weights utility by probability of outcome, whereas in the prospect theory utility or prospect is weighted by a function of probability that reflects the impact on overall value of prospect.

The prospect theory is thus stated as follows

Theorem. *V, the overall value of a regular prospect*[1] *$(x, p; y, q)$, is expressed in terms of the value function v and the weighting function π,*

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$$

where $v(0) = 0$, $\pi(0) = 0$, and $\pi(1) = 1$.

[1] either $p + q < 1$, or $x \geq 0 \geq y$, or $x \leq 0 \leq y$

The authors suggested that the value function v is defined on deviations from the reference point; generally concave for gains and commonly convex for losses; and steeper for losses than for gains. In addition, the weighting function π has the following properties

(1) subadditivity of small p: $\pi(rp) > r\pi(p)$ for $0 < r < 1$
(2) overweighting of small p: $\pi(p) > p$ for small p
(3) subcertainty: $\pi(p) + \pi(1-p) < 1$ for all $0 < p < 1$
(4) subproportionality: $\frac{\pi(pq)}{\pi(p)} \leq \frac{\pi(pqr)}{\pi(pr)}$ for $0 < p, q, r \leq 1$

Subadditivity comes from the fact that $V(600, 0.001) > V(300, 0.002)$ even though $v(600) < 2v(300)$. Overweighting of small p helps explain why people buy lottery and insurance at the same time, where for instance

- lottery: $V(5000, 0.001) > V(5, 1)$
- insurance: $V(-5000, 0.001) < V(-5, 1)$

Subcertainty is based on empirical evidence. In particular, recall the paradox

$$\pi(1)v(1) > \pi(0.1)v(5) + \pi(0.89)v(1) + \pi(0.01)v(0)$$

since $v(0) = 0$, $\pi(0) = 0$, and $\pi(1) = 1$

$$[1 - \pi(0.89)]v(1) > \pi(0.1)v(5)$$

$$\pi(0.1)v(5) + \pi(0.9)v(0) > \pi(0.11)v(1) + \pi(0.89)v(0)$$

or

$$\pi(0.1)v(5) > \pi(0.11)v(1)$$

Therefore, $1 - \pi(0.89) > \pi(0.11)$ and $\pi(0.89) + \pi(0.11) < 1$ Subproportionality is derived from the observation that if $V(x, p) = V(y, pq)$ then $V(x, pr) < V(y, pqr)$. For instance, $V(1000, 0.5) = V(10000, 0.1)$ then $V(1000, 0.05) < V(10000, 0.01)$.

References

Kahneman, Daniel, and Amos Tversky (1979) "Prospect Theory: An Analysis of Decision under Risk", Econometrica, XLVII (1979), 263-291.