

PROBABILITY MODEL FOR BLACKJACK POKER

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The main idea of this paper is to find the results and errors of the probabilistic model that use sampling with replacement for blackjack poker. It starts with analyzing serendipitous hands when a player holds two 9s against 7 from a dealer.

Initial Condition: Player (9,9) VS Dealer (7, x) or (7, x, y); $x \neq 10, 11$

It finds the probability distribution $\mathbb{P}(X = x)$ by assuming that the chance of getting each card is equally likely. Then, it continues on finding the probability distribution $\mathbb{P}(X = x, Y = y)$ for case that the hands terminate after the dealer draws only once and the author writes out a matrix for its conditional sample space and weighed conditional sample space.

After that, the paper explores some conditional probabilities in the first draw events for the case of push VS defer and win VS bust of the dealer. The result is that players will expect to on or before the first draw with the probability only 0.3384 for the serendipitous hand.

Then, the paper starts to calculate the probability in the case of sampling with replacement to compare with the one calculated earlier. It follows the similar process of calculating the conditional sample space, weighed conditional sample space, first draw events. It finds that there is 7.785% errors for $\mathbb{P}(\text{Both defer})$ as compared to the case of sampling without replacement. Moreover, the players expectation to win on or before the first draw is now 0.3432, which is 0.48% too large compared to the case of sampling without replacement. Therefore, the paper concludes that the player may under-predict if he use sample with replacement.

The paper continues on exploring more when the serendipitous hand continues to the 4th dealer card. Now a player stands (9,9) while a dealer draws (7, x, y, z) and the sample space will be 3D with 18,424 events. In this case, the players expectation to win after the second draw has changed to 0.40119, which is significantly different from the first draw expectation.

To make it clearer, the paper explores the case when serendipitous hand continues to the 5th dealer card. Now a player stands: (9,9) while dealer draws (7, x, y, z, w) $DF2\{z, w; z, w = 2, 3, \dots, 10(11, 1)\}$. Players expectation now increases to 0.410396 while the chop factor keeps dropping every time the number of draws increase. This chop factor will eventually converge to zero.

The paper ends with the conclusion that sampling without replacement is more accurate than the

other way because the errors can be significant in the long run.

REFERENCES

Crooke, Charlie H. "Probability Models for Blackjack Poker." ScienceDirect. August 26, 2009.