A SUMMARY OF HARRISON AND LEMOINE'S *ON THE VIRTUAL AND ACTUAL WAITING TIME DISTRIBUTIONS OF A GI/G/1 QUEUE*

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This paper (J. Appl. Prob. 13, 833-836 (1976)) considers a GI/G/1 queue, such that G and H are the distributions for the actual and virtual waiting times, respectively. A G1/G/1 queue is a line with one server at the end such that the distributions of both the arrival times of customers and the serving times are general and unknown. First, the authors develop the same path relationship between the actual and virtual processes that Takacs used in his theorem. This relationship states that the limit as n approaches x, the probability that the waiting time of the n^{th} customer does not exceed x approaches the distribution G evaluated at x. Similarly, as time t approaches ∞, the probability that the waiting time to get served at time t approaches the distribution H evaluated at x. The authors then define two distribution functions $\mathcal{F}$ and $\mathcal{B}$, which are virtual analogs of F and B, where F and B are the distributions of service times and times in between arrival of customers in the actual case. Using these distributions and the above relationship, they give a proof of Hooke's theorem similar to the proof of Takacs' theorem. Hook's theorem states that the probability the virtual waiting time of customer $n + 1$ does not exceed x is equal to the probability that the actual waiting time of customer $n+$ the service time of customer $n-$ the virtual time between the arrivals of customers n and $n - 1$ does not exceed x. Combining these two theorems with Lindley's equality, which gives a similar relationship involving only actual waiting times, and assuming that arrivals are expoentially distributed (this special case is known as an M/G/1 queue), they conceptually derive the Pollaczek-Kinchine formula for the mean waiting time of the Laplace transform of W distribution. A key observation in their analysis is noting that if F and B are exponentially distributed, then so are $\mathcal{F}$ and $\mathcal{B}$. Using the inverse Laplace transform, the actual mean waiting time can be recovered. This result is noteworthy because it does not rely on the imbedded Markov Chain characteristic of waiting times. Thus, when we can assume the above path relationship between the virtual and actual distributions, we can quantify mean waiting times in a more efficient fashion.

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