Math/Stat 341: Probability
First Lecture

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Williams College

Introduction and Objectives

Introduction / Objectives

Probability theory: model the real world, predict likelihood of events.

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Objectives

- Obviously learn probability.
- Emphasize techniques / asking the right questions.
- Model problems and analyze model.
- Elegant solutions vs brute force (parameters in closed form versus numerical solutions).
- Looking at equations and getting a sense: $\log -5$ Method: $\frac{p \pm pq}{p+q \pm 2pq}$.

Types of Problems

- Biology: will a species survive?
- Physics / Chemistry / Number Theory: Random Matrix Theory.
- Gambling: Double-plus-one.
- Economics: Stock market / economy.
- Finance: Monte Carlo integration.
- Marketing: Movie schedules.
- Cryptography: Markov Chain Monte Carlo.
- 8 ever 9 never (bridge).

My (applied) experiences

- Marketing: parameters for linear programming (SilverScreener).
- Data integrity: detecting fraud with Benford's Law (IRS, Iranian elections).
- Sabermetrics: Pythagorean Won-Loss Theorem.

Course Mechanics

Grading / Administrative

- Move at fast pace, responsible for reading before class: 5% of grade. HW: 15%. Writing: 10%. Midterm: 30% (if there are two exams only best counts). 'Final' exam: 40%. You may also do a project for 10% of your grade (which reduces all other categories proportionally).
- Pre-reqs: Calc III, basic combinatorics / set theory, linear algebra.

Office hours / feedback

- When I'm in my office (schedule online), rest TBD.
- Feedback ephsmath@gmail.com, password 1793williams.

Other

- Webpage: numerous handouts, additional comments each day (mix of review and optional advanced material).
- Clickers: see how well we can estimate probabilities, always anonymous.
- Probability Lifesaver: opportunity to help write a book, lots of worked examples.
- Creating HW problems: mix of ones you can solve and ones you want to learn about.
- Gather and analyze some data set of interest.
- PREPARE FOR CLASS! Must do readings before each class.

Being Prepared

Never know when an opportunity presents itself....

S. J. Miller at the Sarnak 61st Dinner

(copyright C. J. Mozzochi, Princeton N.J)

Being Prepared

- Your Job:
  - ◇ Be prepared for class: do reading, think about material.
  - ◇ Come to me, the TAs and each other with questions.
- My/TAs Job:
  - ◇ Provide resources, guiding questions.
  - ◇ Be available.

Other: Advice from Jeff Miller

- Party less than the person next to you.

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Happy to do practice interviews, adjust deadlines....

Gambling

Football Wager

2007: Friend of a favorite student bet $500 at 1000:1 odds on Patriots going undefeated and winning the Superbowl.

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Football Wager

2008: In third quarter, Pats leading, Vegas offers to buy back the bet at 300:1, told no....

WHAT WAS THE BETTOR'S MISTAKE?

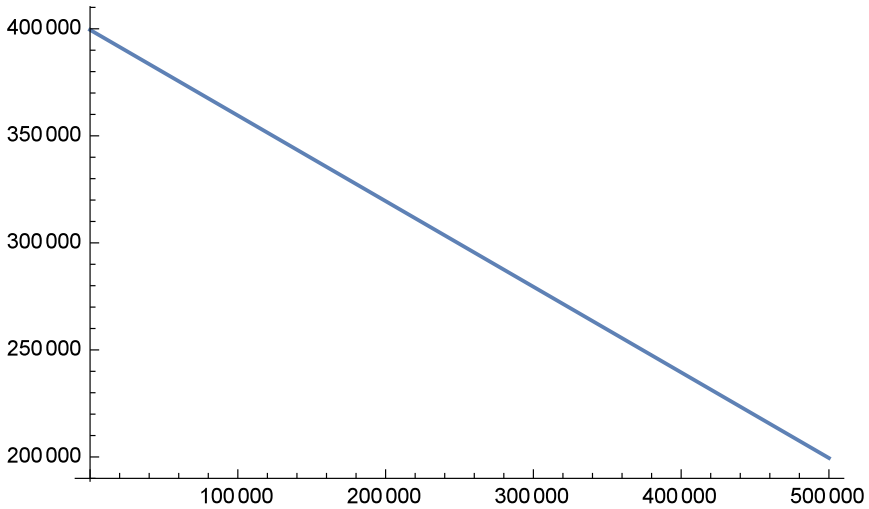
Hedging

Pats win with probability p, Giants $q = 1 - p$.

Bet \$1 bet on Giants, if they win get \$$x$.
Already bet \$500 on Patriots, now bet \$$B$ on the Giants.

Expected Winning:

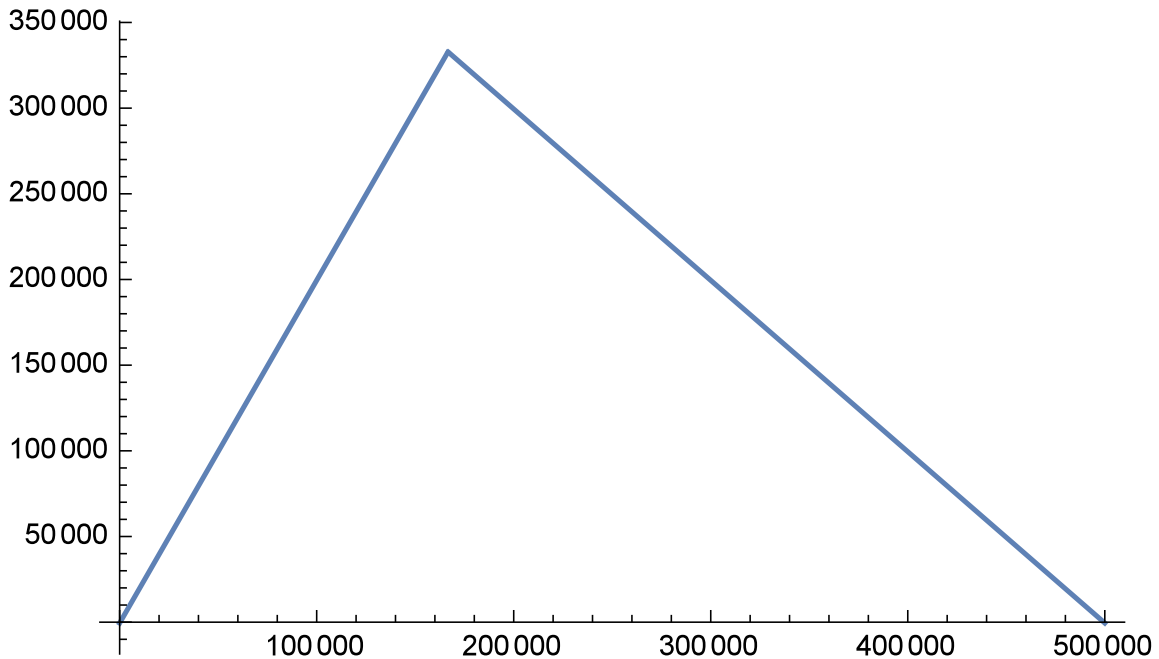
$$f(p, x, B) = p \cdot 500000 + (1 - p)Bx - 500 - B.$$



Guaranteed Winnings

By hedging can ensure some winnings:

$$g(p, x, B) = \min(500000, Bx) - 500 - B.$$

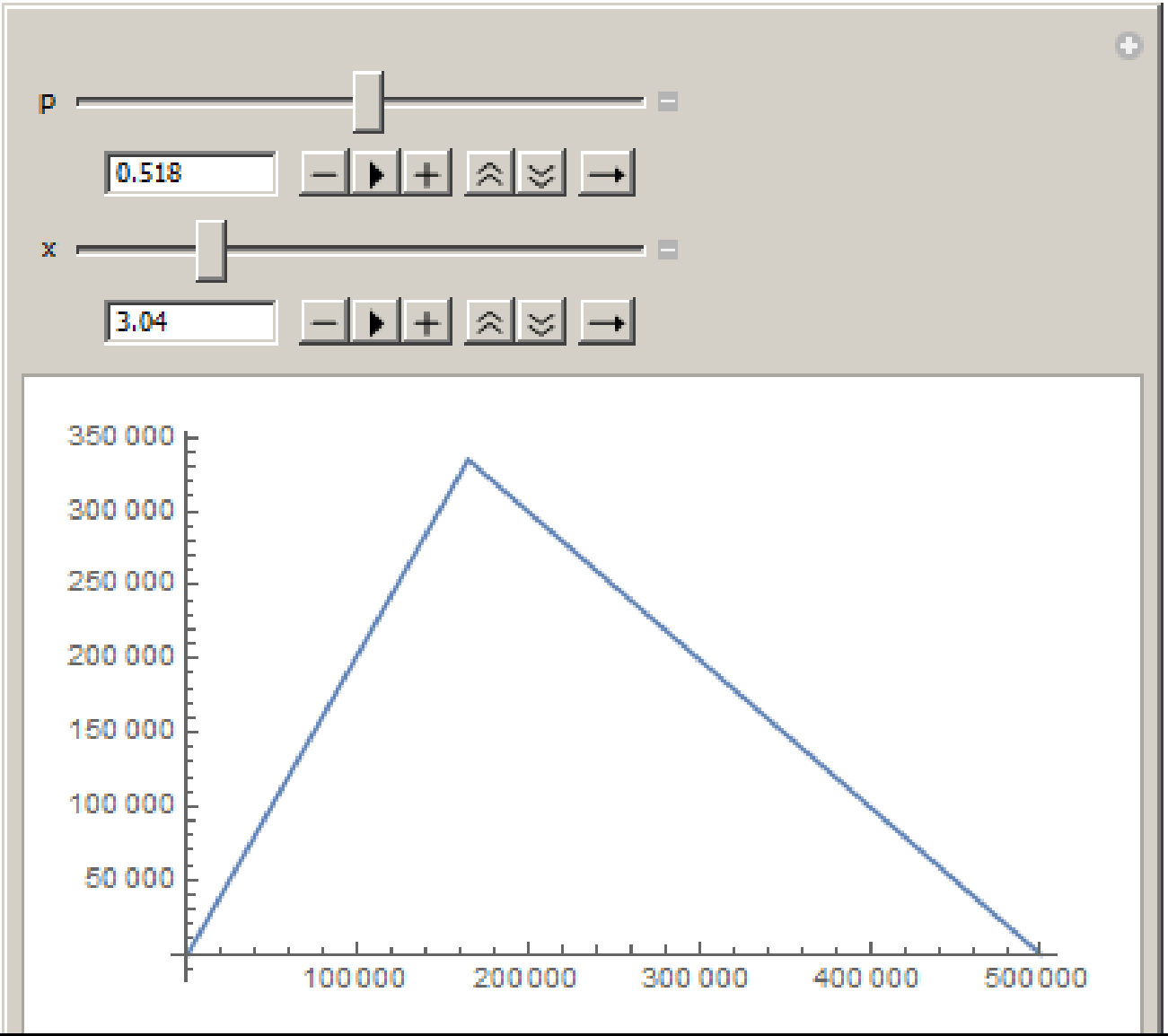


Here $p = .8, x = 3$.

Mathematica Code

```
f[p_, x_, B_] := 500 000 p + (1 - p) B x - 500 - B
g[p_, x_, B_] := Min[500 000, B x] - 500 - B
Plot[f[.8, 3, B], {B, 0, 500 000}]
Plot[g[.8, 3, B], {B, 0, 500 000}]
Manipulate[Plot[g[p, x, B], {B, 0, 500 000}], {p, 0, 1}, {x, 1, 10}]
```

Mathematica Code



Sabermetrics Club at Williams....

http://fivethirtyeight.com/features/

a-head-coach-botched-the-end-of-the-super-bowl-and-it-wasnt-pete-carroll/