14th Annual Green Chicken Contest – 1991

1. Jeremy has ten rods, having lengths 1,2,...,10. How many different ways are there to make a triangle by choosing three appropriate rods?

2. Show that there are infinitely many solutions in positive integers $x < y < z$ for which

$$x!\, y! = z!$$

3. Evaluate $\int_0^{\frac{\pi}{2}} f(x)\, dx$ where

$$f(x) = \frac{\sin^3 x}{\sin^3 x + \cos^3 x}$$

Hint: Consider $f(\frac{\pi}{2} - x)$.

4. Let $1, 3, 5, \ldots, (2n-1)$ be the first n odd numbers. Show that their product is less than n^n.

5. No matter which 1001 distinct positive integers are chosen from $\{1,2,\ldots,1991\}$, prove that two must have difference 9.

6. What is the largest even integer not expressible as the sum of two odd composite numbers? Prove it.