

**The Fifteenth Annual Green Chicken Contest**  
Saturday, 24 October 1992

1. You have 12 coins all of which appear identical, and all but one of which have the same weight. You also have a pair of balances, which will enable you to determine if one collection of objects is heavier than another. What is the minimum number of weighings it will take to find the odd coin out? Explain your answer.

2. Find a four-digit natural number  $N$  such that the last four digits of the product  $1782N$  are 1893 (in that order).

3. Which is bigger:

$$\int_0^1 \cos(x^2) dx \quad \text{or} \quad \int_0^1 \cos(\sqrt{x}) dx?$$

Explain your answer.

4. A positive integer is  $T$ -free if there are no  $T$ 's in its decimal expansion. Does the series

$$\sum_{\substack{n \in \mathbb{N} \\ n \text{ is } T\text{-free}}} \frac{1}{n}$$

converge or diverge? Prove your answer.

5. (a) William Middlebury is an inhabitant of the island of Knights (who always tell the truth) and Knaves (who always lie). Like the island of Manhattan, everyone on this island is either rich or poor. William is surely in love, but the woman of his dreams is particular - she will only marry a rich Knave. What single statement can he make to her to convince her that he has what she's looking for in a man (i.e. that he is indeed a rich Knave)?

(b) Suppose instead she wants a rich Knight. Now what single statement could he make to convince her that he's the perfect mate?

(Pardon the political incorrectness - I stole this from an ancient Green Chicken exam.)

6. It is possible to divide a square into a finite number of smaller squares, all of which are unequal in area. Show that you can't do the same thing with a cube, that is, it is impossible to divide a cube into a finite number of cubes, all of which are unequal in volume. (Hint: Suppose you could. Think about the smallest square on a face of the cube.)