

## The Sixteenth Annual Green Chicken Contest

October 16, 1993

1. Suppose an isosceles triangle has equal sides of length 10 and a base of variable length. What is the length of the third side of the triangle having maximal area?

2. Let  $a_1 = a_2 = 1$  and  $a_{n+1} = \frac{1000a_n + 1000000}{a_{n-1}}$  for  $n \geq 2$ . What is  $a_{1993}$ ?

3. At Middlebury College 7 students registered for American history, 8 students for British history, and 9 students for Chinese history. No student is allowed to take more than one history course at a time. Whenever two students from different classes get together, they decide to drop their current history courses and each add the third. Otherwise there are no adds or drops. Is it possible for all students to end up in the same history course?

4. Let  $f^n(x)$  denote the  $n^{\text{th}}$  power of  $f(x)$ . For a differentiable function  $f(x)$ , describe  $\lim_{h \rightarrow 0} \frac{f^n(x+h) - f^n(x)}{h}$  in terms of  $f(x)$  and  $f'(x)$ .

5. The Green Chicken is filled with black and white marbles and then two marbles are randomly drawn. It is known that the probability of one being black and the other white is  $1/2$ .

(a) Show that the total number of marbles is a perfect square.

(b) Show that the number of white and black marbles are each triangular numbers (i.e. the sum of consecutive integers beginning with 1).

6. Let  $F: \mathbb{R} \rightarrow \mathbb{R}$  be a real-valued function which satisfies the relation  $F(x)F(x+1) + F(x+1) + 1 = 0$  for all  $x$ . Show that  $F$  is not continuous.