# The 17th Annual Green Chicken Contest

Saturday, October 29, 1994

1. In the multiplication problem:

$$\begin{array}{r} \text{GREEN} \\ \times \quad 17 \\ \hline \text{CHICKEN} \end{array}$$

each of the letters G, R, E, N, C, H, I, and K represents a distinct numeral from 0 to 9, and G and C are not allowed to be 0 (so that GREEN is really a 5-digit number and CHICKEN is really a 7-digit number). Find a solution for this.

2. Suppose $x > 1$ is a real number such that $x - \frac{1}{x}$ is an integer. Show that $x^3 - \frac{1}{x^3}$ must be an integer and that $x^2 - \frac{1}{x^2}$ can never be an integer.

3. You have 1000 coins in a row, numbered 1 through 1000, all heads up. On Day 1, you turn over all the coins (they're now all tails up). On Day 2, you turn over every second coin (starting with coin 2, then coin 4, etc.). On Day 3, you turn over every third coin (starting with coin 3, then coin 6, etc). You continue in this manner until Day 1000, when you just turn over coin # 1000. After Day 1000, which coins are tails up?

4. Suppose $f(x)$, $g(x)$ are differentiable functions for $x > 0$ satisfying

$$f'(x) = -\frac{g(x)}{x}, \qquad g'(x) = -\frac{f(x)}{x}.$$

Characterize all such $f, g$.

5. Consider the function $f(m) = \int_1^m \frac{e^{-mx}}{x} dx$, where $m > 0$. Find the value of $m$ which maximizes $f$.

6. Define $b_n$ to be the Fibonacci-style sequence

$$1, 3, 4, 7, 11, 18, \ldots$$

which is constructed recursively by $b_1 = 1$, $b_2 = 3$, and $b_{n+2} = b_{n+1} + b_n$. Define a new sequence $a_n = \frac{b_n}{3^n}$.

a. Show that the series $\sum_{n=1}^{\infty} a_n = \frac{1}{3} + \frac{3}{9} + \frac{4}{27} + \frac{7}{81} + \cdots$ is a convergent series.

b. Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$.