

**20th Green Chicken Contest (November 8, 1997)**

1. In a large urn there are 75 white balls and 150 black balls. Next to the urn is a large pile of black balls. The following procedure is performed repeatedly: Draw two balls at random from the urn.

(a) If they are both black, then one of them is put back in the urn and the other is thrown away.

(b) If one is black and one is white, then the white one is put back in the urn and the black one is thrown away.

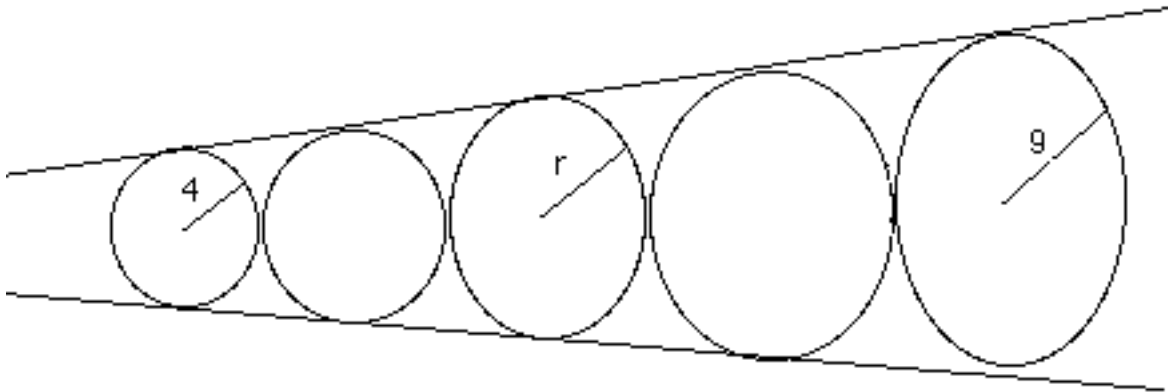
(c) If they are both white, then they are both thrown away and a black ball from the pile is put in the urn.

After each turn there is one less ball in the urn. Eventually the urn will contain just one ball. What color is this last ball?

2. Does any row of Pascal's triangle have three consecutive entries that are in the ratio 1:2:3? (Recall that for  $n \geq 0$ , the  $n$ th row of Pascal's triangle contains the entries

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad \text{for } 0 \leq r \leq n$$

3. Five circles are tangent to each other and a pair of lines as shown below. Find  $r$  (the radius of the middle circle.)



4. A college dormitory has 250 students. For every pair of students, A and B, there is a language that A speaks that B does not, and a language that B speaks that A does not. What is the smallest total number of languages that could be known by the students?

5. The mean and standard deviation of any set of seven consecutive integers are both integers. Find the next positive integer  $n$  such that the mean,  $\bar{x}$ , and standard deviation,  $s$ , of  $n$  consecutive integers are both integers .

6. Show that a given geometric square can be decomposed into  $n$  squares, not necessarily all the same size, for  $n = 4$  and all  $n \geq 6$ .