The Twenty First Annual Green Chicken Contest

October 31, 1998

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1. Suppose you have a sphere of radius R and four planes that are all tangent to the sphere such that they form an arbitrary tetrahedron (it can be irregular). What is the ratio of the volume of the tetrahedron to its surface area?

2. Let a, b, and c be positive numbers. Prove that a/b + b/c + c/a >= 3.

3. Six boxes are numbered 1 through 6. How many ways are there to put 20 identical balls into these boxes so that none of them is empty?

4. For which real numbers c is there a straight line that intersects the curve y = x^4 + 9x^3 + cx^2 + 9x + 4 in four distinct points?

5. Calculate the following sum 1(1!) + 2(2!) + 3(3!) + . . . + n(n!).

6. Let f(x) be a function which is not identically equal to zero and f(x-y) = f(x)f(y) for all x and y. Find f(x).