The Twenty Second Annual Green Chicken Contest

October 30, 1999

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1. Three students are chosen randomly. Is it more likely that at least two were born on the same day of the week or that none of them were born on a weekend?

2. Four travelers must traverse a narrow bridge at night. At most two of them can be on the bridge at the same time. There is only one flashlight which is needed to cross, but it can be shared. The travelers can cross the bridge in one minute, two minutes, five minutes, and ten minutes respectively (pairs cannot go faster than the slower traveler). What is the least amount of time required for all four travelers to cross the bridge safely?

3. A quadrilateral has one vertex on each side of a unit square. Show that the lengths a, b, c, d of the sides of the quadrilateral satisfy the inequalities

$$2 <= a^2+b^2+c^2+d^2 <= 4$$

4. (a) Show that $2(\text{sqrt}(n+1) - \text{sqrt}(n)) < 1/\text{sqrt}(n) < 2(\text{sqrt}(n) - \text{sqrt}(n-1))$ for all $n >= 1$.

(b) Let S = "the sum from n=1 to n=2500 of terms of the form 1/sqrt(n)" (you might first write this using "sigma notation" to make it clearer!). Find [S], the greatest integer less than or equal to S.

5. The points of a plane are colored either red or blue. Prove that one of these colors contains pairs of points at every possible mutual distance.

6. An integer is powerful if each of its prime factors occurs to the second power or more. For example, $1800 = 2^3 * 3^2 * 5^2$ is powerful, while $1785 =(3)(5)(7)(17)$ is not. (One point for noting the significance of the numbers above.) Prove that there are an infinite number of pairs of consecutive powerful numbers.