

26th Green Chicken Contest - 2003

1. John and Amy each drive round trip from Greentown to Chickenville and immediately back. John drives 60 miles per hour one way and 40 miles per hour the other way. Amy drives at a constant speed of 50 miles per hour both ways and arrives back in Greentown 6 minutes ahead of John. How far apart are the two towns?

2. Among 27 identical-looking coins, one is counterfeit weighing slightly less than the others. Using a double pan balance, identify the counterfeit coin in as few weighings as possible.

3. (a) Find a sequence $\{a_n\}$ with $a_n > 0$ for all $n \geq 1$ such that the series $\sum \frac{a_n}{n^2 \sqrt{n}}$ and $\sum \frac{1}{a_n}$ both converge.

(b) Prove that there does not exist a sequence $\{b_n\}$ with $b_n > 0$ for all $n \geq 1$ such that the series $\sum \frac{b_n}{n^2}$ and $\sum \frac{1}{b_n}$ both converge.

4. A boat has sprung a leak. Water continues to come in at a uniform rate and some has already accumulated when the leak is discovered. At this point, 12 people of equal ability can pump the boat dry in 3 hours, while 5 people require 10 hours. How many people are needed to pump the boat dry in 2 hours?

5. What is the sum of the solutions of the equation $\sqrt[4]{x} = \frac{12}{7 - \sqrt[4]{x}}$?

6. Given any set of 2003 integers, show that for some pair of them either their sum or their difference is divisible by 4002.