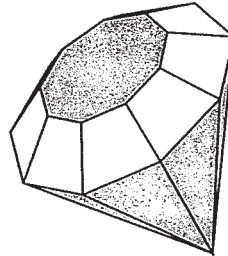
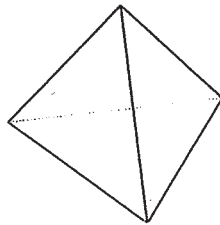


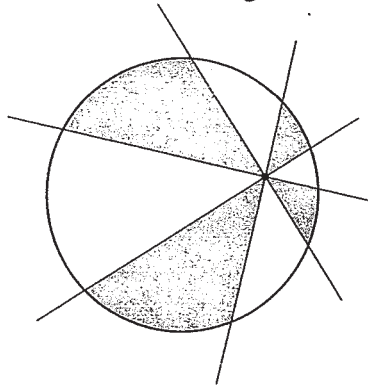
Twenty-Eighth Annual Green Chicken Contest

October 15, 2005

1. Show that $n! + 2005$ is not a perfect square for any whole number $n \geq 10.15$.
2. Suppose that you have a fair 4-sided die and a fair 9-sided top. Show that whole numbers can be put on the 13 available spots on these objects in such a way that that when they are rolled and spun (respectively), the sums that result are distributed just as they would be for a pair of standard six-sided dice.



3. Suppose that some point on a perfectly round pizza is chosen, and that the pizza is cut into 8 slices by cutting at 45° angles through that point. If we color the slices of the pizza alternately blue and purple, show that the total blue area and the total purple area will be equal, independent of the point chosen and the angle of the first cut.



4. Consider the series $\sum_{n=0}^{\infty} \frac{(-1)^n (2n+2)}{(2n+3)!}$.
 - (a) Prove that this series converges.
 - (b) Find its exact value.
5. Consider the graph $y = f(x)$ of some differentiable function f on the real line. Show that if $(a, f(a))$ is the point on this graph closest to the origin, then $a + f'(a) \cdot f(a) = 0$.
6. Find all triples (a, b, c) of whole numbers for which $ab + 1$ is a multiple of c , $bc + 1$ is a multiple of a , and $ca + 1$ is a multiple of b .