Counting number of edges, thickness, and chromatic number of $k$-visibility graphs

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April 6, 2013
Bar Visibility Graphs

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Bar Visibility Graph

Bar 1-Visibility Graph

Bar Visibility Representation

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Thickness and Chromatic Number

**Definition**

The *thickness* $\Theta(G)$ of a graph $G$ is the least number of colors needed to color the edges of $G$ so that no two edges with the same color intersect.

**Definition**

The *chromatic number* $\chi(G)$ of a graph $G$ is the least number of colors needed to color the vertices of $G$ so that no two vertices with the same color are adjacent.
Upper Bound on Thickness of Bar $k$-Visibility Graphs

**Theorem**

$\Theta(G_k) \leq 6k$ for all bar $k$-visibility graphs $G_k$.

- Great improvement over old quadratic bound of $18k^2 - 2k$ found by Dean et al. (2005).
- Found with method used to bound thickness of semi bar 1-visibility graphs, found by Felsner and Massow (2008).
- Not tight: $\Theta(G_1) \leq 4$ proven by Dean et al. (2005).
- There exist $G_k$ with $\Theta(G_k) \geq k + 1$.
- Maximal thickness grows at $O(k)$.
Proof of Upper Bound

- Based on bound of $\chi(G_k) = 6k + 6$ by Dean et al. (2005).
- Method: construct graph based on representation. Thicken bars to rectangles. Assume no two vertices have same $x$-coordinate.
- Use one-bend edges.
Proof of Upper Bound

- No two horizontal or two vertical segments intersect.
- Color edges based on vertex-coloring of $G_{k-1}$.
- Intersecting edges intersect in rectangle of horizontal segment, thus left endpoints of the edges must have different colors when considering $(k - 1)$-visibility.
Semi Bar Visibility Graphs

Semi Bar Visibility Graph

Semi Bar Visibility Representation

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Semi Bar Visibility Graphs

Semi Bar Visibility Graph  Semi Bar Visibility Representation

Semi Bar 1-Visibility Graph  Semi Bar 1-Visibility Representation

Counting number of edges, thickness, and chromatic number of \( k \)-visibility graphs
Upper Bound on Thickness of Semi Bar $k$-Visibility Graphs

**Theorem**

$\Theta(G_k) \leq 2k$ for all semi bar $k$-visibility graphs $G_k$.

- Better than bound found using $\chi(G_k) \leq 2k + 3$, found by Felsner and Massow (2008)
- Proof based on how many one-edges cross any given bar.
- There exist $G_k$ with $\Theta(G_k) \geq \left\lceil \frac{2}{3}(k + 1) \right\rceil$
- Maximal thickness grows at $O(k)$. 

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Arc Visibility Graphs

Arc Visibility Graph

Arc Visibility Representation

Counting number of edges, thickness, and chromatic number of $k$-visibility graphs
Arc Visibility Graphs

Arc Visibility Graph

Arc 1-Visibility Graph

Counting number of edges, thickness, and chromatic number of $k$-visibility graphs
Theorem

Arc $k$-visibility graphs with $n$ vertices have at most $(k + 1)(3n - k - 2)$ edges.

- Found by considering endpoints of arcs
Theorem

Arc $k$-visibility graphs with $n$ vertices have at most $(k + 1)(3n - k - 2)$ edges.
- Found by considering endpoints of arcs

Theorem

$\chi(G_k) \leq 6k + 6$ for all arc $k$-visibility graphs $G_k$.
- Bounded by maximum number of edges
Upper Bound on Thickness of Rectangle $k$-Visibility Graphs

**Theorem**

$\Theta(G_k) \leq 12k$ for all rectangle $k$-visibility graphs $G_k$.

- Double the upper bound for bar $k$-visibility graphs.
Conclusion

What Did We Do?

- Improved bounds on thickness of bar $k$-visibility graphs, created bound on thickness of semi bar $k$-visibility graphs
- Placed bounds on number of edges and chromatic number of arc $k$-visibility graphs
- Found bound on thickness of rectangle $k$-visibility graphs
Conclusion

Future work:

- Tighten bounds for bar, semi bar, arc, rectangle $k$-visibility graphs

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Acknowledgements

- Jesse Geneson
- Dr. Tanya Khovanova, MIT Math Dept.
- Dr. John Rickert
- CEE, RSI, MIT
- Mr. Regan, Mr. Beebee, Mr. Cheng, Department of Defense
- Williams College

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Bar 1-Visibility Representation of $K_8$

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