Introduction to the Taylor-Socolar Tile

Reflections of the tile are allowed, and matching rules enforced by decorations as shown:
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The second version of the tile’s matching rules are enforced by shape alone:

However, this tile is no longer a connected set.

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1) Have matching rules enforced by shape alone
2) Be a simply connected set
3) Tile the plane with rotations alone, and not reflections
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2) Be a simply connected set

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It’s important to note that 1) and 2) together create a strong condition for the allowed matching rules. Together, they limit the matching rules to local, pairwise matches.
Controversy

- Obviously, the TS tile breaks either 1) or 2), depending on which version of the tile we are examining.
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- Taylor and Socolar’s defense: the ”shape alone” condition can be broken in a manner that isn’t too objectionable.
- Relaxing 1) and instead requiring that all matching rules enforce a pairwise interaction, that still seems acceptable.
It is possible however to make a 3D version of the tile which tiles the 2D plane aperiodically, is simply connected, and forces the tiling by shape alone.

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3 Harris, Edmund ”Socolar and Taylor’s Aperiodic Tile”. *Maxwell’s Demon, Vain Attempts to Construct Order*. April 1, 2010.
Proof of Aperiodicity

The tile’s matching rules quickly imply that adjacent tiles must form small triangles made from the black lines at the top and bottom of the tile:
Further, the flags ensure that every small black triangle must have another small triangle across from it like so:
Proof of Aperiodicity

This forces the tiling to form honeycomb like lattices:

These are 2 level 1 lattices. Note that the center of every lattice is a free tile.
Proof of Aperiodicity

Suppose we fill the right free tile with a tile that is rotated 60 degrees counter clockwise from a vertical orientation like so:
Proof of Aperiodicity

Then the matching rules immediately constrain the nearby tiles (shown here by adding the additional decorations) and start to form larger triangles:
Proof of Aperiodicity

The flags again force a large triangle across from every large triangle. This gives us a level 2 lattice:
Proof of Aperiodicity

The center of a level two lattice is a free tile as well. Suppose we choose a vertically oriented tile for the center of this lattice:
Proof of Aperiodicity

- The same as before, each larger triangle forces an equal sized triangle across from it, forming a level 3 lattice.
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- Applying the previous argument iteratively, we know that we will have a free tile at the center of a level $n$ lattice, which will create bigger triangles which forces a level $n+1$ lattice.
Proof of Aperiodicity

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- Applying the previous argument iteratively, we know that we will have a free tile at the center of a level \( n \) lattice, which will create bigger triangles which forces a level \( n+1 \) lattice.
- Since there is no biggest triangle nor lattice, any tiling of the TS tile will be non periodic. Then, the tile forms an aperiodic tiling set.