Optimization of an Archimedean Screw

Alison Shappy
Pat Knapp
Julie Mullowney
Colin Delaney

Saint Michael's College

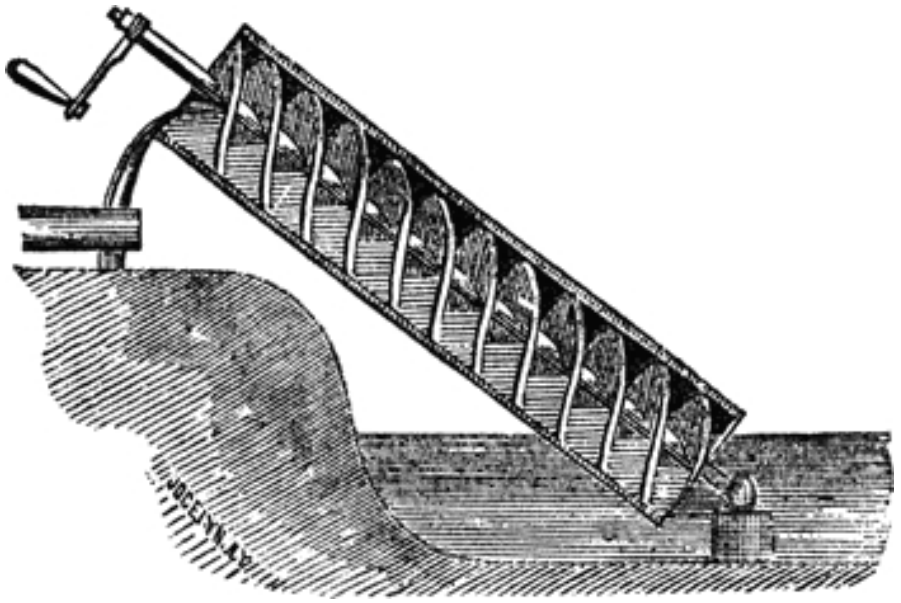
Basic Function: How it Works

- Used to transport water and other materials up the length of the screw by rotating the screw on an inclined plane.

http://youtu.be/5gq3Vm4vifU

History

- Invented around first century BC by Archimedes
- First detailed description of the screw was found written by Vitruvius (85BC – 20BC)
- Crankshafts were added to screws in 16th century by Konrad Kyeser



Rorres FIG. 2

Terms

- Pitch (Λ)– length of the screw from peak to peak for one cycle.
- $K = \tan\theta$, which gives the screw incline
- Chute – Area between adjacent blades, and inner and outer radii
- Bucket – one of the areas occupies by the trapped water within any one chute.

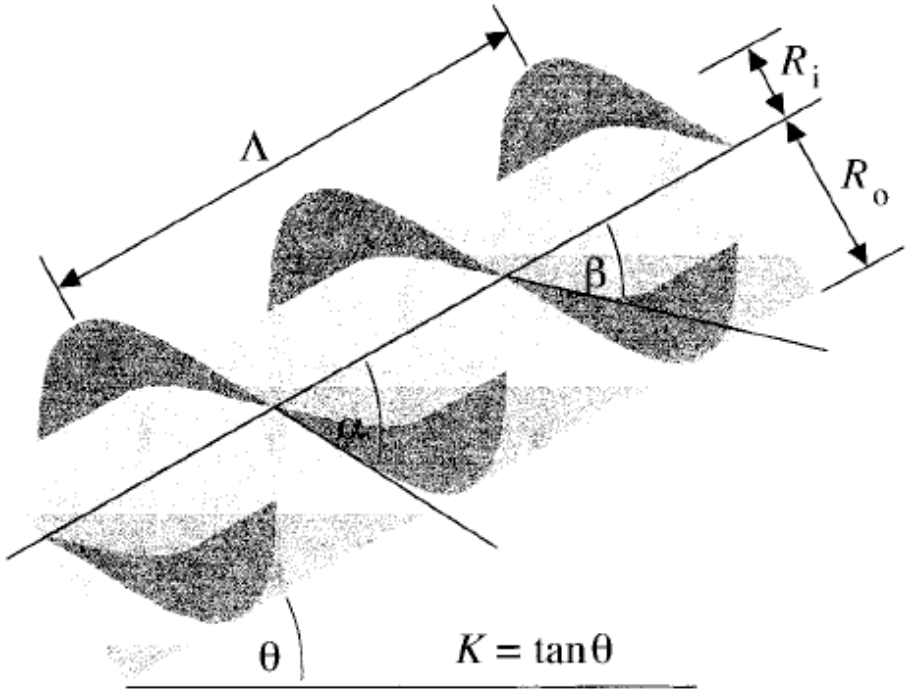
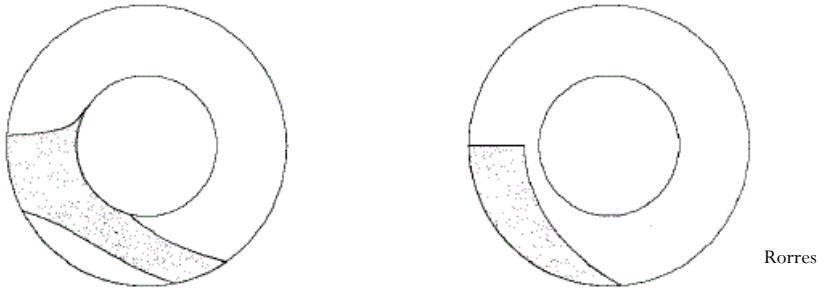


FIG. 5. Profile View of Segment of Two-Bladed Archimedes Screw

Rorres

Terms (continued)

- $v = V_T / \pi R_o^2 \Lambda$, the volume ratio
- $p = R_i / R_o$, the radius ratio
- $\lambda = K\Lambda / 2\pi R_o$, $0 \le \Lambda \le 2\pi R_o / K$, the pitch ratio



(a) $\Lambda \in (0, 2\pi R_i/K]$ (b) $\Lambda \in (2\pi R_i/K, 2\pi R_o/K]$

Our design

Goal: Maximize the amount of water carried for each turn of the screw.

External Parameters:

- R_o = radius of screw's outer cylinder
- L = total length of screw
- K = slope of screw (dimensionless)

Internal Parameters:

- R_i = radius of screw's inner cylinder
- Λ = pitch (or period) of one blade
- N = number of blades

Our Parameters:

Number of Blades: 2
Screw Length : 5"
Outer cylinder radius: .5"
Angle of incline: 30°

Optimal Ratios

Using the given independent variables the optimal ratios for a two bladed screw can be calculated:

- radius ratio (p^*) is 0.5369
- pitch ratio (λ^*) is 0.1863
- volume ratio ($v(N, p^*, \lambda^*)$) is 0.2747

Optimization for Buckets

Optimizing the inner radius (R_i*)

$R_i^* = p^* R_o$ where p* is the value of p that maximizes $\lambda v(N, p, \lambda)$

Optimizing the pitch (Λ*)

$\Lambda^* = 2\pi R_o \lambda^* / K$ where λ^* is the value of λ that maximizes $\lambda v(N, p, \lambda)$

Equation for optimizing a bucket:

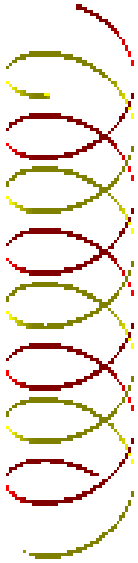
$$V_T^* = (2\pi^2 R_o^3 / K) * \lambda^* v(N, p^*, \lambda^*)$$

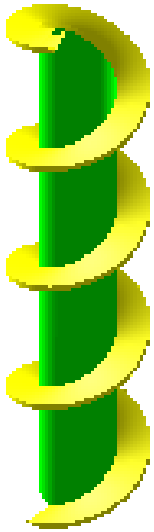
Using the optimal ratios for the pitch, radius, and volume

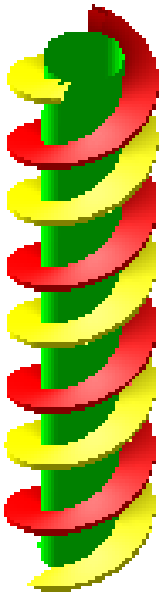
$R_i^* = .26845$”

$\Lambda^* = 1.0137$”

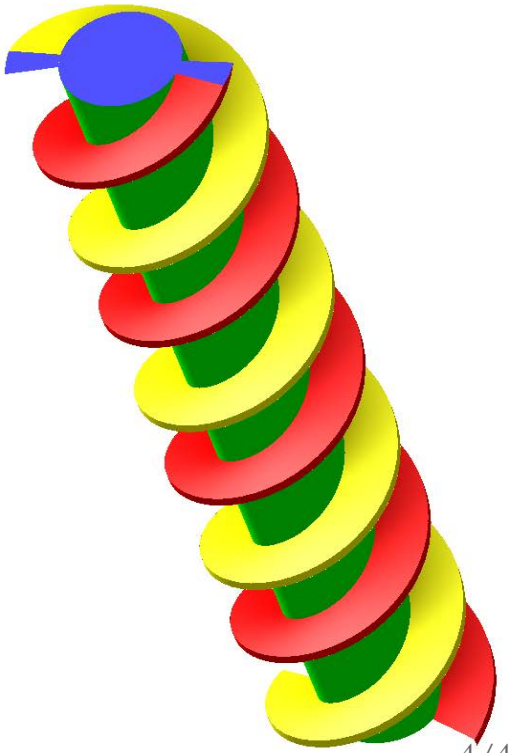
$V_T^* = .2188$ -inches^3







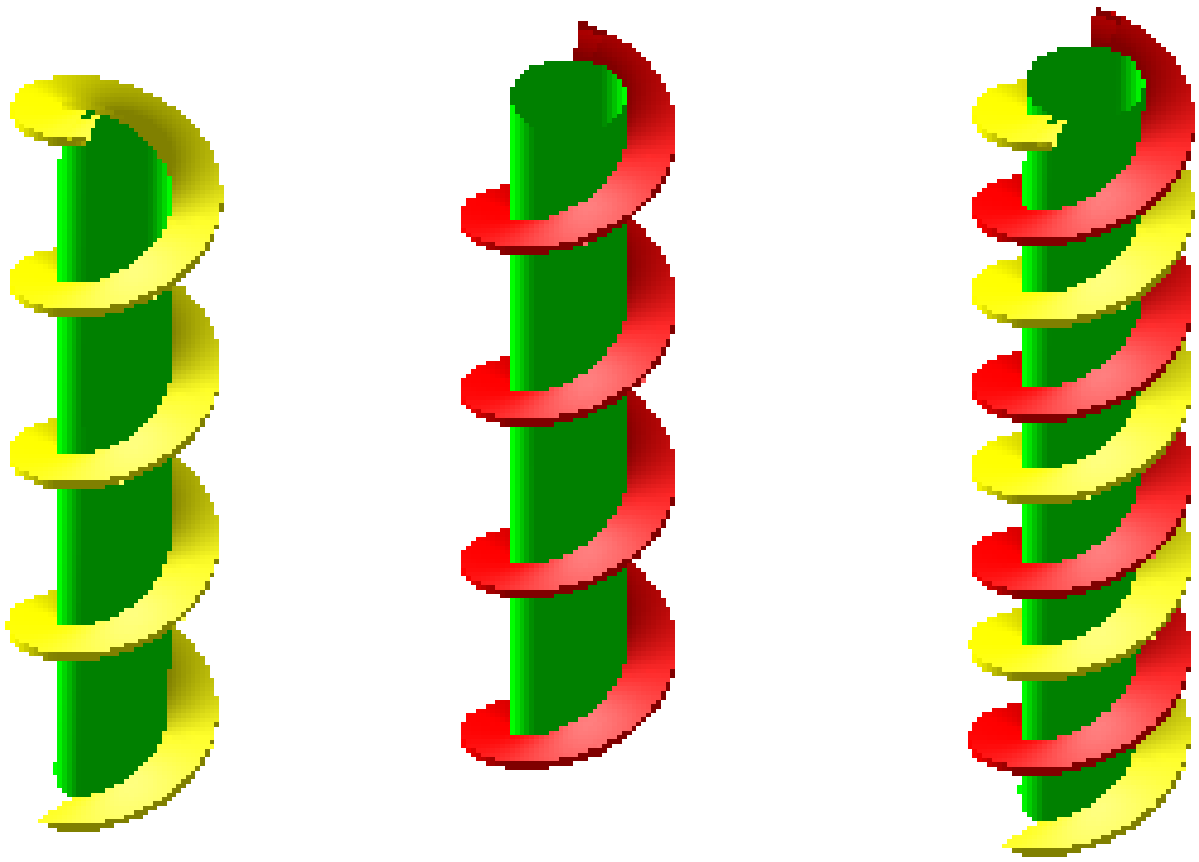
3D Modeling



3D Modeling

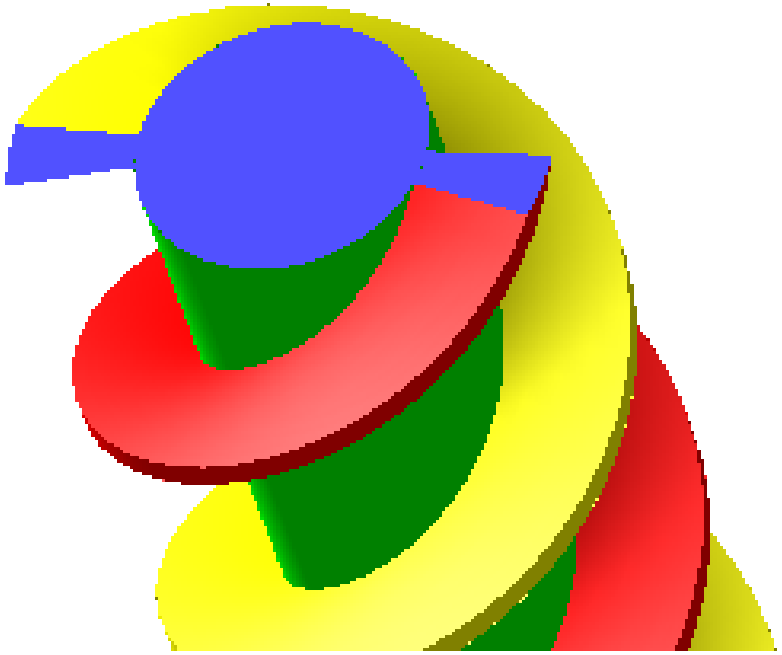
```
> T1 := [r·cos(pitch·t), r·sin(pitch·t), t] : #top helicoid
> B1 := [r·cos(pitch·(t + w)), r·sin(pitch·(t + w)), t] : #bottom helicoid
> O1 := [rend*cos(pitch*t), rend*sin(pitch*t), max((1-s)*(t - w), 0) + min(s*t, tend)] : #fill the T1-B1 chute
> top1 := plot3d(T1, r = rstart..rend, t = 0..tend, color = yellow, grid = [20, 200]) :
> bottom1 := plot3d(B1, r = rstart..rend, t = 0..tend, color = yellow, grid = [20, 200]) :
> thick1 := plot3d(O1, s = 0..1, t = 0..tend + w, color = yellow, grid = [20, 200]) :
> T2 := [r·cos(pitch·(t + adj)), r·sin(pitch·(t + adj)), t] :
> B2 := [r·cos(pitch·(t + adj + w)), r·sin(pitch·(t + adj + w)), t] :
> O2 := [rend*cos(pitch*(t)), rend*sin(pitch*(t)), max((1-s)*(t - w - adj), 0) + min(s*(t - adj), tend)] :
> top2 := plot3d(T2, r = rstart..rend, t = 0..tend, color = red, grid = [20, 200]) :
> bottom2 := plot3d(B2, r = rstart..rend, t = 0..tend, color = red, grid = [20, 200]) :
> thick2 := plot3d(O2, s = 0..1, t = adj..tend + adj + w, color = red, grid = [20, 200]) :
```

3D Modeling



```
> C1 := [rstart·cos(pitch·(t)), rstart·sin(pitch·(t)), (1-s)*max(t - Pi - w, 0) + s*min((t - w), tend)] :
> shaft1 := plot3d(C1, s = 0..1, t = w..tend, color = green, grid = [20, 200]) :
> C2 := [rstart*cos(pitch·t), rstart*sin(pitch·t), (1-s)*max(t - Pi - w, 0) + s*min((t - w + Pi), tend)] :
> shaft2 := plot3d(C2, s = 0..1, t = adj..tend, color = green, grid = [20, 200]) :
```

3D Modeling



```
> Cap1 := plot3d([r*cos(t), r*sin(t), 0], t=0..2*Pi, r=0...26845, color=blue):
> Cap2 := plot3d([r*cos(t), r*sin(t), 5], t=0..2*Pi, r=0...26845, color=blue):
> Cap3 := plot3d([r*cos(pitch·t), r*sin(pitch·t), 5], t=5..5+w, r=0.26845..0.5, color=blue):
> Cap4 := plot3d([r*cos(pitch·(t+Pi)), r*sin(pitch·(t+Pi)), 5], t=5..5+w, r=0.26845..0.5, color=blue):
> Cap5 := plot3d([r*cos(pitch·t), r*sin(pitch·t), 0], t=0..w, r=0.26845..0.5, color=blue):
> Cap6 := plot3d([r*cos(pitch·(t+Pi)), r*sin(pitch·(t+Pi)), 0], t=0..w, r=0.26845..0.5, color=blue):
```

Archimedean Screw Used Today

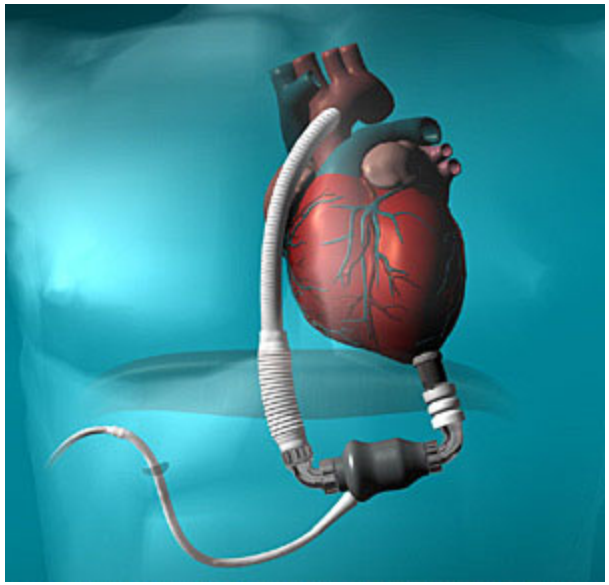
- Sewage treatment: used to drain containers of sewage and to transport them into treatment tanks
- Fishing nurseries: allows both water and fish to be transfers safely
- Hydroelectric power: water turns the screw and the screw turns the generator creating power
- Farming: used horizontally to harvest plants and feed them into a container, called a "screw conveyor"

http://talybontenergy.co.uk/wp-content/uploads/2010/09/Osbaston+Screw+Turbine.jpg

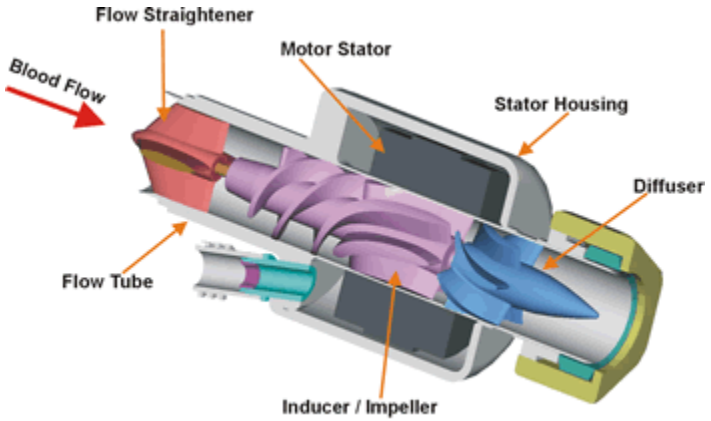
http://upload.wikimedia.org/wikipedia/commons/e/e1/John_Deere_combine_and_tractor_at_work.jpg

Applications (continued)

- Medicine: rotor sends blood through the body using the continuous rotation of a screw.



http://texasheart.org/Research/images/hm2_illustration-thoratec12-03s.jpg



http://www.cts.usc.edu/graphics/debakey-vad-pump-schematic.gif

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