

Leading, learning and herding*

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Abstract

We analyze a game theoretic model of social learning about a consumption good with endogenous timing and heterogeneous accuracy of private information. We show that if individuals value their reputation for the degree to which they are informed, this reduces the incentive to learn by observing others and exacerbates the incentive to consume the good before others, i.e., to attempt to be an “opinion leader.” Consequently, reputation concerns reduce the average delay of consumption of new goods, and increase (reduce) the probability of herding on consumption (non-consumption).

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1 Introduction

The seminal models of social learning and herding (Bikhchandani, Hirshleifer, and Welch (1992), Banerjee (1992)), and many of the models that followed, assume that the order in which individuals take actions is exogenously determined. This assumption is not innocuous. In fact, the premise of this literature—that people gain information by observing the actions of others—makes exogenous timing immediately questionable. If there is a benefit to waiting and observing others, why would anyone want to act first?

On the other hand, there is strong empirical evidence that some people actually prefer to try new things before their peers. That is, some people want to be *opinion leaders*, *early adopters*, or *market mavens*, despite this causing them to lose valuable information they would have gained by waiting and observing others. See, e.g., Feick and Price (1987) for a discussion of this type of behavior and numerous examples of its occurrence. To elaborate on one particular example, there is evidence consumers prefer to be first to see new movies, as box office sales are on average highest in the first week of release and decline thereafter (?). But consumers do indeed learn about how good movies are by waiting and observing their success (Moretti (2008)).¹

This paper analyzes a very simple game theoretic model that reconciles social learning and opinion leader behavior. The motivation is both to better understand these conflicting forces, and to gain insight into how their presence may affect social outcomes via rational herding. Our *a priori* hypothesis was that incorporating opinion leader incentives would weaken the informational content of publicly observable actions, which would increase the chances of inefficient herding. Our results provide some support for this hypothesis, but turn out to be somewhat more complex. We also introduce a new refinement concept for Perfect Bayesian Equilibrium, which may have applications in other settings.

In the model, two consumers decide whether to buy a new product whose quality is

¹If the timing of movie-going was mostly driven by learning motives, we would expect ticket sales for a successful movie to increase over time initially after release, since only the most informed consumers would be willing to see it at first. That is, if time after release and cumulative ticket sales were plotted against each other, the “diffusion curve” would take the well known “S” shape (see e.g. Kapur (1995)). While this does happen for movies occasionally, it is relatively rare, as the average curve appears uniformly concave (?).

unknown. Both can either buy immediately, or wait and have the option to buy after observing their peer’s decision, so the order of actions is endogenous. The consumers have different abilities to obtain and process a private signal about the product’s quality. We show that in equilibrium, if consumers only care about utility from consumption, then a consumer buys the product immediately only if her information indicates the product is high quality and is sufficiently accurate such that waiting cannot convince her to change her mind, as long as she is at least marginally impatient. A consumer waits if her information is less accurate, or indicates the good is low quality. Consequently, if a consumer cares about her reputation for the degree to which she is informed—in addition to direct consumption utility—this increases the incentive to buy immediately, increasing average consumption in the initial period, for two reasons. First, buying immediately signals high accuracy of private information. Second, and more subtly, reputation concerns reduce the incentive to wait, learn and imitate one’s peer, since this would signal *low* accuracy of private information. Since these effects interact and reinforce each other, the tradeoff between reputation and social learning is exacerbated.

To examine the effects of the tradeoff on rational herding, we extend the model in a simple way and find that reputation concerns have different effects on the formation of different types of information cascades and herd behaviors. When reputation concerns are low, consumers sometimes delay consumption even when their private information indicates the product is high quality in order to wait and learn. This is observationally equivalent to their peers to not buying due to having private information indicating the product’s quality is poor, so consuming and not consuming are asymmetric, from a learning perspective. This causes herding on non-consumption to be more likely, as compared to when the order of actions is exogenous. When reputation becomes more important, herding on consumption becomes more likely. Although both the incentives to consume immediately and to abstain from consumption later are increased, and these have competing effects on the probability of herding on consumption, the first effect is dominant.

An increased probability of herding on consumption is socially beneficial when the product is high quality, and harmful otherwise. The magnitudes of the effects in both cases are approximately the same, and so changes in the level of reputation concerns cause socially beneficial and harmful herding effects that approximately cancel, assuming the product is

equally likely to be good or bad. Thus, within our model, introducing reputation concerns is neither socially beneficial nor harmful. We discuss directions for future research in the paper’s final section.

1.1 Related Literature

There is a substantial literature on social learning with endogenous timing, but most of it assumes a key reason to take action sooner rather than later is to avoid an exogenous cost of delay.² Our paper can be thought of as endogenizing this cost (reputation loss) driving early action. Moreover, in situations in which it is ambiguous whether or not reputation is driving delay costs, our paper suggests a comparative static for determining this: if reputation is a key factor, then people more concerned with reputation (such as “the new guy” whose reputation is relatively uncertain and malleable) will be more likely to consume early, and/or to avoid herding late. We do not formalize this prediction, but it follows directly from the model.

However, even in the absence of heterogeneity in reputation concerns, reputation loss and other costs of delay do not have equivalent behavioral effects. To see this, suppose the alternative to reputational costs of delay is pure impatience. Very impatient consumers who do not act immediately are willing to learn from and imitate earlier-acting peers, as the costs of impatience become sunk. Reputation-concerned consumers are unwilling to learn and imitate, since reputation is always variable. There are almost certainly other insights that can be gained by incorporating reputation in social learning models; we consider our model just a first step in this direction.

While there is some literature on opinion leaders,³ we are not aware of literature on the opinion leader/social learning tradeoff we identify. The literature on bandwagon effects, e.g. Corneo and Jeanne (1997) is related, but only indirectly since it does not focus on information. The literature on reputation and herding focuses on incentives to conform, e.g. Scharfstein and Stein (1990), and not on the effects of reputation concerns on the order of actions. See ? for an analysis of leadership in organizations, which also does not address

²See, e.g., ?, Gul and Lundholm (1995), ?, and SgROI (2003).

³See Kircher and Postlewaite (2008) in particular. Bikhchandani, Hirshleifer, and Welch (1998) briefly discuss this phenomenon, which they refer to as “fashion leaders,” but do not discuss how it would be affected by reputation concerns or refer to any formal analysis.

reputation.

We refer to both herds and information cascades in our paper; see Smith and Sørensen (2000) or Çelen and Kariv (2004) for discussion of the distinction between these terms.⁴ We analyze cascades (and thus herds) because they are naturally of interest, have been a focus of much of the related literature, and the quality of decisions made by herds provides a natural measure of social welfare. The informational asymmetry of consumption and non-consumption—essentially that consumption can only signal good news, while non-consumption signals either no news or bad news—plays an important role in our paper. ? studied this asymmetry, showing it surprisingly implies that products with larger customer bases are less likely to have herds form that purchase the product.⁵ The literature on endogenous timing in oligopoly (e.g., ?) is also related, but only indirectly.

2 The Model

The model is a two player, two period game. Before the first period a new product with unknown quality $\theta \in \{G, B\}$ (good/bad) is released. The prior is common and for simplicity: $Pr(\theta = G) = 0.5$. Each player i has a private signal on quality, $s_i \in \{g, b\}$, with $Pr(s_i = g|\theta = G) = Pr(s_i = b|\theta = B) = \pi_i \geq 0.5$. There is common knowledge that π_2 is equal to a particular value, denoted π . That is, player 2 (P2, a she) has a “secure” reputation, in that both players know her signal’s informativeness, and know they both know it. Player 1 (P1, a he) is “insecure”: he knows π_1 , but P2 does not know it and has the prior $\pi_1 \sim U[\underline{\pi}, \bar{\pi}]$, with $0.5 \leq \underline{\pi} < \bar{\pi} \leq 1$.⁶ Since the parameter π_i represents i ’s ability to collect and process information, it is assumed to be desirable to have others think it is higher (i.e., to have a better reputation). We assume $\pi = E(\pi_1) = 0.5(\underline{\pi} + \bar{\pi})$ so P2 is as informed as P1 on average.

⁴The latter write, “When acting in a herd, individuals choose the same action, but they may have acted differently from one another if the realization of their private signals had been different. In an informational cascade, an individual considers it optimal to follow the behavior of her predecessors without regard to her private signal since her belief is so strongly held that no signal can outweigh it. Thus, an informational cascade implies a herd but a herd is not necessarily the result of an informational cascade.”

⁵? studied another type of asymmetry in social learning models—that sellers of a unit good can only be made worse off by the formation of a herd—since, given the lack of supply, only herding on non-consumption can occur.

⁶The term insecure, as commonly used, sometimes refers to situations like this, but other times refers to situations in which a person does not know his own ability.

Each player chooses whether to consume the product in period $t = 1$, and also in $t = 2$ if he/she chose not to consume in $t = 1$. That is, consumption only occurs once or not at all; this is a standard unit demand assumption.⁷ Both consumers' actions in both periods are publicly observable. Let I_{it} denote the information available to player i at the start of period t . So, I_{i1} includes i 's signal and if $i = 1$ the realization of π_1 ; I_{i2} includes I_{i1} and the history of both players' actions, for $i = 1, 2$. P1's objective is to maximize $u(x_1|\theta) + \alpha E_2(\pi_1)$, with $E_2(\pi_1)$ denoting P2's posterior (to period 2) expectation of P1's ability. The parameter α represents the importance of reputation, with $\alpha \geq 0$. P2's objective is just to maximize her expectation of $u(x_2|\theta)$, since her reputation is secure. We write $x_i = 1$ if i consumes the product in either period and $x_i = 0$ otherwise. The utility function is normalized so that $u(x_i = 1|\theta = G) = 1$, $u(x_i = 1|\theta = B) = -1$ and $u(x_i = 0|\theta) = 0$. Let x_{it} denote player i 's action in period t ; $x_{it}=0/1$ for not consume/consume.

We assume that if in $t = 1$ either player's expected payoff from consuming in that period is equal to her/his expected payoff from waiting (and making the optimal decision to consume/not consume in $t = 2$), then the player consumes in $t = 1$. This assumption serves the purpose of assuming both players are at least marginally impatient. For simplicity, and because the focus of the analysis is on how other forces affect the order of actions, we do not model impatience explicitly, since as discussed above this has been the focus of previous literature. But it is natural to think that consuming "now" is preferable to consuming "later." This assumption is only important for the $\alpha = 0$ case. For consistency and without loss of generality we also assume if either player (who has not yet consumed) is indifferent between consuming and not consuming in $t = 2$, then the player consumes.

The timing of the game is summarized as follows. 1) Nature chooses θ , π_1 , s_1 and s_2 , the product is released, P1 privately observes π_1 and both players observe their s 's; 2) $t = 1$: each player i chooses x_{i1} and then observes x_{-i1} ; 3) $t = 2$: each player i chooses x_{i2} if $x_{i1} = 0$ and then observes x_{-i2} ; 4) P2 updates beliefs about π_1 and both players obtain their payoffs.

⁷Zhang (2007) refers to this as "one-sided commitment" and gives the example of the consumer decision of whether or not to buy a new computer operating system (once it is purchased, the consumer is typically stuck with it, and does not buy another in the short-term future); another example is the decision of whether to go to a movie opening weekend or wait until a later date, since people usually do not go to the same movie more than once.

We note that our model involves three extensions, and one (major) simplification to a benchmark social learning model, which Bikhchandani, Hirshleifer, and Welch (1992) describe as a special case of their general model, and typically used in experimental tests of social learning.⁸ In this model (henceforth the BHW model) a sequence of *ex ante* identical individuals make a binary decision. The order in which they act is exogenous, and each individual has one private binary signal and observes the decisions of any individuals who act before her/him. The simplification we make to this set-up is that we drop all but two of the decision-makers. Our extensions are: 1) the accuracy of one individual's signal is his private information; 2) this individual may care about his reputation for accuracy; and 3) the order in which the two individuals act is endogenous. Further comments on the model are provided in the final section.

3 Analysis

Let $\hat{\pi}_1$ denote the realization of π_1 . Let $x_{it}^*(I_{it})$ denote the equilibrium strategy of player i in period t , a function of the available information. It is convenient to sometimes just write a particular component of I_{it} that is of interest, for example we sometimes write $x_{i1}^*(s_i)$. We use the Perfect Bayesian Equilibrium solution concept (PBE).

It is helpful to first look at the case of $\alpha = 0$, and recall that $\pi = E[\pi_1]$.⁹

Proposition 3.1. *If $\alpha = 0$, the unique PBE is characterized as follows.*

1. *P1's strategy is:*

- a) *If $\hat{\pi}_1 \geq \pi$, then $x_{11}^*(g) = 1$, $x_{11}^*(b) = 0$, and $x_{12}^*(I_{12}) = 0$ for all I_{12} .*
- b) *If $\hat{\pi}_1 < \pi$, then $x_{11}^*(s_1) = 0$ for all s_1 , and $x_{12}^*(I_{12}) = 1$ if and only if $x_{21} = 1 \in I_{12}$.*

2. *P2's strategy is:*

- $x_{21}^*(g) = 1$, $x_{21}^*(b) = 0$, and $x_{22}^*(I_{22}) = 1$ if and only if $x_{21} = 0, x_{11} = 1 \in I_{22}$.*

3. *Out-of-equilibrium beliefs for both players are unrestricted.*

⁸Weizsacker (forthcoming) provides a recent meta-study of data from 13 distinct experiments of this type.

⁹It might be ideal to simply refer the reader to a paper from the endogenous timing literature for a characterization of equilibrium for this case. However, even when $\alpha = 0$ our model is distinct from other endogenous timing models, as far as we aware, due to our model's assumption that the signal accuracy is a continuous variable that is private information for just one player.

Proof. Showing P1's strategy is optimal is simple. In case 1.a, his signal is weakly more informative than P2's, thus P1's optimal action is unaffected by waiting and observing P2's action in $t = 1$, and so P1 consumes if and only if $s_1 = g$ and only in $t = 1$ ("early"), given the tie-breaker assumption for cases where expected utility from consumption is the same in both periods. Case 1.b is the opposite: P1's signal is less informative than P2's, and since P1 knows P2 reveals her signal by her $t = 1$ action, P1 clearly has an incentive to wait until $t = 2$ ("late") and then mimic P2's $t = 1$ action.

Regarding P2's strategy, it is clear that if $s_2 = b$ and she has no other information, her expected utility from consumption is negative and so $x_{21} = 0$ is optimal. If P2 then observes $x_{11} = 1$, P2 will infer that $s_1 = g$ and that P1's information is better in expectation ($E(\pi_1|x_{11} = 1) > \pi$), so it will be optimal for P2 to choose $x_{22} = 1$. If $s_2 = g$, expected utility from consumption in $t = 1$ is positive, but it is possible P2 could benefit from waiting and observing x_{11} . This would only be true if observing $x_{11} = 0$ would cause P2's expected utility from consumption to become negative. We verify this is not the case by showing P2's belief that $\theta = G$ after observing $x_{11} = 0$ is still greater than 0.5:

$$\begin{aligned} Pr(\theta = G|x_{11} = 0, s_2 = g) &= \frac{Pr(x_{11} = 0|\theta = G, s_2 = g)Pr(\theta = G|s_2 = g)}{Pr(x_{11} = 0|s_2 = g)} \geq 0.5 \\ &= \frac{[\int_{\underline{\pi}}^{\pi} f(\pi_1)d\pi_1 + \int_{\pi}^{\bar{\pi}} (1 - \pi_1)f(\pi_1)d\pi_1]\pi}{[\int_{\underline{\pi}}^{\pi} f(\pi_1)d\pi_1 + \int_{\pi}^{\bar{\pi}} (1 - \pi_1)f(\pi_1)d\pi_1]\pi + [\int_{\underline{\pi}}^{\pi} f(\pi_1)d\pi_1 + \int_{\pi}^{\bar{\pi}} \pi_1 f(\pi_1)d\pi_1](1 - \pi)} \geq 0.5 \\ &\Leftrightarrow -0.625\underline{\pi}^2 + \underline{\pi}(0.5 + 0.25\bar{\pi}) + (0.375\bar{\pi} - 0.5)\bar{\pi} \geq 0. \quad (1) \end{aligned}$$

A lower-bound for the left-hand side is obtained when $\underline{\pi}$ is at its minimum, 0.5, or maximum, $\bar{\pi}$. When $\underline{\pi} = 0.5$, the left-hand side is strictly positive given $\bar{\pi} \in (0.5, 1]$. When $\underline{\pi} = \bar{\pi}$, the left-hand side is zero, implying it is strictly positive if $\underline{\pi} < \bar{\pi}$.

It is possible there exists an equilibrium in which P2 does not follow a good signal in $t = 1$ (i.e., $x_{21}^*(g) = 0$), since P2 could learn from x_{11} it is optimal to not consume. In this case, P1 would never be able to learn from P2's $t = 1$ action, so P1 would follow his signal in $t = 1$ for all $\hat{\pi}_1$ (again, due to the tie-breaker assumption). However, this would not cause P2's observation of x_{11} to change her optimal action when $s_2 = g$ since $E(\pi_1|x_{11}) = \pi$ for all x_{11} in this case, and so even if $x_{11} = 0$ this would cause abstaining from consumption to be

strictly optimal. Thus, P2 would have no incentive to wait and learn if $s_2 = g$, which implies that P2 always does prefer to follow her signal in $t = 1$, which causes P1 to prefer to wait if $s_1 = g$ and $\hat{\pi}_1 < \pi$. This proves the equilibrium's uniqueness.

Out-of-equilibrium beliefs are unrestricted because the only actions that occur outside equilibrium are consumption in the second period after neither player consumed in the first period. But the beliefs of either player after observing this action do not affect either player's payoff, so they can take any value.

□

This result implies both players are potentially opinion leaders and potentially observational learners who imitate the first mover. P1 is a leader when $s_1 = g$ and is more accurate than s_2 , as in this case he cannot learn useful information from waiting (i.e. waiting cannot convince him to not consume), so he may as well consume as soon as possible. P2 is a leader when $s_2 = g$ because waiting cannot bring her useful information either. This is because for waiting to convince her to not consume, she would have to know both that $s_1 = b$ and s_1 is accurate. This never occurs because P1 only reveals that s_1 is accurate when $s_1 = g$. This is due to the inherent asymmetry of consumption actions; consuming early is unambiguous as it can only occur when the player's signal is g , while not consuming early can either result from a bad signal or an inaccurate signal. As a result, learning only goes in one direction, in a sense, for P2. P2 only imitates P1 when P2's signal is b , P1's is g , and $\hat{\pi}_1 \geq \pi$. P2 never imitates when $s_2 = g$. P1 can learn both to consume and not consume from P2 (i.e., P1 can learn whether s_1 is g or b) as long as $\hat{\pi}_1 < \pi$.

The proposition provides an explanation for the timing of actions without invoking reputation concerns. However, it is clear that since players who consume early are better informed on average, if players do value their reputation for the degree to which they are informed, this will increase the incentive to be a first mover—and mitigate the incentive to be a social learner. In this case, the beliefs of P2 after observing an unexpected action by P1 will affect P1's incentives. We focus on a class of PBE with off-equilibrium-path beliefs restricted in a particular way, which we argue is natural, defined as follows.

Definition 3.1. *A Reputational Equilibrium is a PBE in which: 1) $x_{12}^*(I_{12}) = 0$ if $\{x_{11} =$*

$0, x_{21} = 0\} \subset I_{12}$; 2) $x_{11}^*(I_{11}) = 1$ iff $s_1 = g \in I_{11}$ and $\hat{\pi}_1 \geq \gamma > \underline{\pi}$; 3) $E_2(\pi_1 | x_{11} = 0, x_{21} = 0, x_{12} = 1) \leq \gamma$.

In a Reputational Equilibrium (hereafter RE), P1 never consumes late if both players abstained early (part 1). Part 2 says in equilibrium P1 does consume early if his signal is good and sufficiently accurate (greater than some threshold γ). Part 3 describes off-equilibrium-path beliefs: if P1 consumes late after both players abstained early, then P2 takes a skeptical view of P1's accuracy, $\hat{\pi}_1$ —that it is not greater than γ . The logic is that P1 has the least disincentive to take the off-path action of consuming late when his expected utility from consumption is highest. In general, this is true when $s_1 = g$ and $\hat{\pi}_1$ is as high as possible. But if $s_1 = g$ and $\hat{\pi}_1 \geq \gamma$, then P1 should have consumed early. Since he did not, it is natural for P2 to infer that if $s_1 = g$ then $\hat{\pi}_1 < \gamma$. Moreover, if $s_1 = b$, then P1's incentive to consume would be highest when $\hat{\pi}_1$ is as low as possible, i.e. $\underline{\pi}$, which is also consistent with the RE off-path restriction.

RE is a refinement of PBE in the spirit of the Intuitive Criterion (Cho and Kreps (1987)) and other PBE refinements in which it is generally assumed that when forming beliefs about the type of a deviating player, the non-deviating player places more weight on types with lower disincentives to deviate. Our innovation is to exploit the dynamic nature of the game, so that when P1 deviates in the second period, P2 restricts attention to P1's types consistent with P1's first period action, since that action is always on the equilibrium path. We are not aware of other refinements that restrict off-path beliefs this way. We use RE instead of other refinements (e.g., the Intuitive Criterion) both because RE makes the analysis tractable, and it seems more realistic. It would be implausible for P2 to think P1 has a very informative signal if he consumed late but not early, given that P1's equilibrium strategy is to consume early when his signal is highly informative.

The PBE of Proposition 3.1 is indeed an RE, with $\gamma = \underline{\pi}$. The RE we characterize below with $\alpha > 0$ is thus the natural extension of the unique PBE with $\alpha = 0$.

Proposition 3.2. *If α is sufficiently small but strictly positive, then there exists an RE characterized as follows.*

1. P1's strategy is characterized by two thresholds, $\gamma_1^* \in (\underline{\pi}, \pi)$ and $\gamma_2^* \in (\underline{\pi}, \pi)$, such that:

- a) if $\hat{\pi}_1 > \max\{\gamma_1^*, \gamma_2^*\}$, then $x_{11}^*(g) = 1$, $x_{11}^*(b) = 0$, $x_{12}^*(I_{12}) = 0$ for all I_{12} ;
- b) if $\gamma_2^* \geq \gamma_1^*$ and $\gamma_2^* \geq \hat{\pi}_1 \geq \gamma_1^*$, then $x_{11}^*(g) = 1$, $x_{11}^*(b) = 0$, $x_{12}^*(I_{12}) = 1$ if and only if $x_{11} = 0, x_{21} = 1 \in I_{12}$;
- c) if $\gamma_1^* > \gamma_2^*$ and $\gamma_1^* = \hat{\pi}_1$, then $x_{11}^*(g) = 1$, $x_{11}^*(b) = 0$, $x_{12}^*(I_{12}) = 0$ for all I_{12} ;
- d) if $\gamma_1^* > \gamma_2^*$ and $\gamma_1^* > \hat{\pi}_1 > \gamma_2^*$, then $x_{11}^*(s_1) = 0$ for all s_1 , $x_{12}^*(I_{12}) = 1$ if and only if $s_1 = g, x_{21} = 1 \in I_{12}$;
- e) otherwise $x_{11}^*(s_1) = 0$ for all s_1 and $x_{12}^*(I_{12}) = 1$ if and only if $x_{21} = 1 \in I_{12}$.
2. P2's strategy is $x_{21}^*(g) = 1$, $x_{21}^*(b) = 0$, and $x_{22}^*(I_{22}) = 1$ if and only if $x_{21} = 0, x_{11} = 1 \in I_{22}$, and beliefs off the equilibrium path are $E_2(\pi_1 | x_{11} = 0, x_{21} = 0, x_{12} = 1) = \rho$ for any $\rho \leq \gamma_1^*$.

The proof is in Appendix A; it is complicated but the intuition is fairly straightforward. The proposition says that, just as in the $\alpha = 0$ case, P1 consumes early if his signal is good and $\hat{\pi}_1$ is sufficiently high, and never consumes early, while imitating P2's early action regardless of his signal, if $\hat{\pi}_1$ is sufficiently low. However, while in the $\alpha = 0$ case there was one threshold for $\hat{\pi}_1$ for both types of behavior, π , in the $\alpha > 0$ case there are two thresholds and they are both strictly less than π . The first threshold, γ_1^* , determines whether P1 can be an opinion leader, and the second, γ_2^* , determines whether P1 will *refuse* to be a social learner when $s_1 = b$. P1 may refuse to be a social learner, i.e. refuse to act on information inferred from P2's action that makes expected direct utility from consumption positive, because imitation signals that P1 has inaccurate private information. Since both γ_1^* and γ_2^* are strictly less than π , individuals with insecure reputations are strictly more likely to be opinion leaders, and to avoid social learning, than insecure individuals unconcerned with reputation. Thus average consumption in $t = 1$ is higher with $\alpha > 0$ than $\alpha = 0$. The threshold γ_1^* is the analog to the generic parameter γ used in the definition of RE.

It may be unclear in what sense γ_1^* and γ_2^* are chosen by P1; they are indeed chosen, as they define the optimal strategy for P1. However, they are equilibrium values, so they are known by P2, thus they are only optimal for P1 given that P2 believes that they are being used by P1. It should also be noted that the proposition is an existence result, and does not specify the range of α , and relationship between α and the γ^* 's precisely. We determine the

relationship between both γ_1^* and γ_2^* and α numerically, and present the results in Figure 1. They are unique, for fixed α , although this is not shown in the proof, so there is no question of whether the players will be able to coordinate on particular values. The figure shows how, as implied by Proposition 3.1, they are both equal to π when $\alpha = 0$ and both decrease as α increases. It also shows that γ_1^* is always less than γ_2^* . Intuitively, an increase in α causes γ_1^* to decrease for two reasons—an increased incentive to signal high π_1 in $t = 1$ and decreased incentive to wait, learn and act in $t = 2$. But only the latter incentive applies to γ_2^* , so it is not driven down as “quickly” by increasing α as γ_1^* .

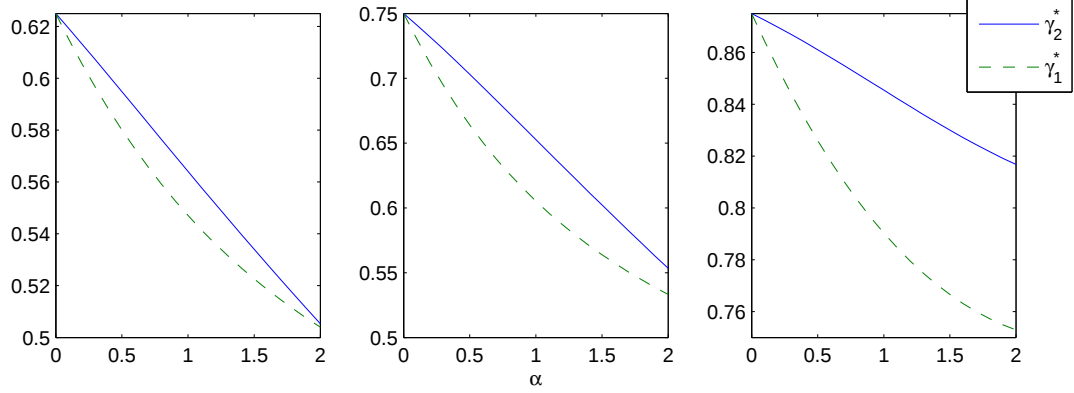


Figure 1: Equilibrium thresholds for P1: $\pi = 0.625$ (left); $\pi = 0.75$ (center); $\pi = 0.875$ (right).

We do not formally characterize equilibria for large α , but it is clear that as reputation concerns increase, P1’s incentives to be an opinion leader, and to not be a social learner, both increase. In fact, if α is sufficiently high P1 will never be a social learner, since the benefit from learning is limited while the reputation cost is increasing in α . Thus, for large α P1 will either consume in $t = 1$ or not at all. High types will follow their signal in $t = 1$, since they are confident in its accuracy, and if their signal is correct following the signal improves reputation (since P2 follows her signal too). While some low types may either consume in $t = 1$ for all s_1 or not consume for all s_1 , either sort of pooling behavior can only happen for a limited fraction of types in equilibrium, as otherwise either consuming in $t = 1$ or not at all will signal P1 is a high type, given reasonable off-path beliefs (like those discussed above). In other words, P1 will likely follow his signal in $t = 1$ for almost all $\hat{\pi}_1$ and not consume in $t = 2$

for all $\hat{\pi}_1$. This behavior is very similar to the behavior in the equilibrium we characterize with relatively large α , as shown in Figure 1 (i.e., both γ^* 's are close to $\underline{\pi}$ when $\alpha = 2$, especially in the left graph).

4 Application to Herding

We next address the implications of these results for the quality of decisions made by larger groups of people. We do this by examining the effect of endogenous order and reputation concerns on what Anderson and Holt (1997) refer to as a reverse cascade—a sequence of actions causing each subsequent consumer to herd on a suboptimal decision: either buying a low quality product, or not buying a high quality product. As discussed at the very end of section 2, our model can be viewed as an adaptation of the BHW model of social learning. It would be difficult to extend our model to allow for a large set of players with reputation concerns and endogenous order. A simpler way to extend our model that allows us to analyze an infinite sequence of decision-makers is to assume the BHW model starts when our model ends. That is, we assume that after P1 and P2 interact in the two-period game described above, an infinite sequence of decision-makers decides whether to consume the product. Each decision-maker is *ex ante* identical, has a private signal with accuracy π , and has perfect information about all the decisions made before him and is fully informed about the equilibrium of the game involving P1 and P2. The equilibrium of this game is not affected by the added presence of the later decision-makers if we make the somewhat restrictive assumption that P1 only values his reputation in the eyes of P2. In other words, we concatenate the BHW model to the end of our model.

Numerical results for various values of π and α with both endogenous (concatenated model) and exogenous (BHW model only) order are provided in Figure 2. We present results for two variants of the exogenous order model; one in which players take each action with 50% probability when indifferent, and one in which players follow their own signal when indifferent between actions.¹⁰ The various signal sequences leading to cascades, which underlie the

¹⁰Bikhchandani, Hirshleifer, and Welch (1992) make the former assumption, but the latter seems empirically more likely (see, e.g., Weizsacker (forthcoming)). Our assumption that P1/P2 consume when indifferent with is still without loss of generality with $\alpha > 0$ because then P2 is never indifferent,

probabilities depicted in the graphs, are presented in Appendix B. The first thing to note from the figure is that endogenizing the order of actions has ambiguous effects on the reverse cascade probabilities, as compared to the probabilities with exogenous order. The direction of the effect depends on the assumption for how players behave when indifferent (endogenous order is on average beneficial when compared to exogenous order with randomization, and slightly harmful compared to exogenous order without randomization). Endogenous order is worse than the latter type of exogenous order as endogenous order increases the probability of a reverse cascade on non-consumption, since P1's incentive to wait and learn causes him to often not reveal his signal even when $s_1 = g$. This result may be sensitive to the assumptions on the binary action/signal space and number of players.¹¹

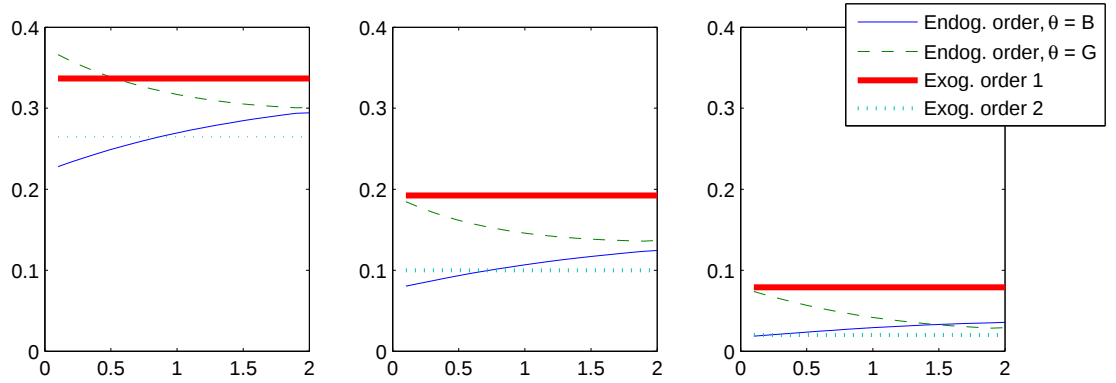


Figure 2: Reverse cascade probabilities (herding on consumption when $\theta = B$ and herding on non-consumption when $\theta = G$): $\pi = 0.625$ (left); $\pi = 0.75$ (center); $\pi = 0.875$ (right). “Exog. order 1” = players randomize actions when indifferent; “Exog. order 2” = players follow own signal when indifferent. Both exog. order probabilities are independent of θ .

The figure also shows that with exogenous order the probability of herding on the wrong decision is the same whether $\theta = G$ or $\theta = B$, but with endogenous order the reverse cascade probability increases as α increases when $\theta = B$, but is decreasing in α when $\theta = G$. A reverse cascade is least likely when order is endogenous, $\theta = B$ and α is low, because herding

and P1 is indifferent with probability zero (only for a measure-zero set of realizations of π_1).

¹¹It is worth noting though that these results show how the conclusion of ?—that endogenous timing generally improves welfare—may be sensitive to his assumption that players have the option to “wait” in each period, which is distinct from the other actions. Also, the exogenous order probabilities would be even higher than those with randomization if we assumed players conformed to the action played by the majority of their predecessors when indifferent.

on consumption in this case requires either $\hat{\pi}_1 \geq \gamma_1^*$ and $s_1 = g$, or $s_2 = g$ and $\hat{\pi}_1 \leq \gamma_2^*$. But the former event is highly unlikely, since when $\hat{\pi}_1$ is higher, it is less likely for s_1 to be incorrect. Also, while with exogenous order P2 consuming followed by P1 consuming starts herding on consumption, with endogenous order it does not—if the third decision-maker gets a bad signal he strictly prefers to not consume with endogenous order. This decision-maker knows that his bad signal nullifies the good signal of P2, and that P1 consumes late for all s_1 if $\hat{\pi}_1 < \gamma_1^*$ but only if $s_1 = b$ if $\hat{\pi}_1 \in [\gamma_1^*, \gamma_2^*]$. The probability of the former event occurring increases as α increases, because this causes γ_1^* to decrease, which makes it less informative to others when P1 consumes in $t = 1$. This is why incorrect herding on consumption becomes more likely as α increases. With endogenous order reverse cascades are most likely when $\theta = G$ and α is low because herding on non-consumption is more likely when the incentive of P1 to wait and learn from the action of P2 is stronger, since this causes P1 to often not reveal his signal, which is observationally equivalent to P1 having a bad signal. The incentive to learn and copy P2's action declines as α increases, which makes herding on non-consumption less likely as α increases.

The bottom line to these results is that reputation is a double-edged sword. When reputation is more important, people consume early and attempt to be opinion leaders more often. This increases (decreases) the probability of incorrect herding on (non)-consumption, which is socially harmful (beneficial). These beneficial and harmful effects appear to more or less cancel out, as Figure 2 indicates the overall (unconditional on θ) endogenous order reverse cascade probability is approximately constant for all α . However, it is unambiguous that increasing reputation concerns cause herding on consumption to be more likely, for all θ .

5 Concluding Remarks

We offer a signaling explanation for why individuals may take an action with limited information, even though by waiting they may be able to make more informed decisions. Empirical evidence shows adoption of new trends often seems too fast to be explained by pure learning dynamics (see footnote 1); our model provides an explanation for this faster speed of adoption. The higher reputation concerns are, the more likely it is both consumers will buy the

new product in the initial period.

We now offer a few remarks on questionable aspects of the model and avenues for future research. First, it may be useful to generalize the RE refinement. Regarding the model, we assume only one of the two initial consumers is insecure only for simplicity; it should not matter if both consumers in the basic model had uncertain reputations. Similarly, it should not matter if the distribution of π_1 were single-peaked rather than uniform. If there were more than two players with uncertain reputations involved in the endogenous order game, this would increase the expected π for the most informed player, increasing the value of waiting and learning to other players. On the other hand, more players would be unwilling to mimic the consumption of earlier moving peers, as compared to the late moving players in the concatenated BHW model we examined. It is thus unclear how changing this would affect the comparative statics for α ; it may cause increasing α to have different and possibly more harmful effects on social learning outcomes.

Another modeling assumption that stands out is the lack of direct communication. It may be unrealistic to think of two acquaintances observing whether or not they go to a movie on opening weekend, but not discussing whether they thought the movie was good or not (though the assumption is more reasonable for products whose use is more likely to be publicly observable, such as a cell phone or clothing item). Our primary reason for not incorporating direct communication in the model is that we wanted to modify the BHW model in a limited way, so the effects of the modifications could be easily seen. However, we are skeptical that allowing direct communication would change the results drastically regardless, if other appropriate changes were made to the model as well. We suspect, empirically at least, individuals who attempt to be opinion leaders may bias their messages to attempt to convince others to follow their actions, even if the early consumers do not enjoy the product that much. This would cause the information value of the direct communication from early consumers to be low. Of course, there would be no point to lying and saying a product was good when it was in fact bad if the followers would realize this upon consuming the product themselves. The followers may not realize the product's quality after consuming it, however, if there is heterogeneity in ability to *evaluate* a product's quality. We are not aware of any economics papers that model this type of heterogeneity, and think it may be worthy of future

research.¹²

Finally, we note we are not fully satisfied with our model's prediction that people who consume new products before their peers are generally better informed (although the difference becomes small as α becomes large). We conjecture that a model with different assumptions may make the opposite prediction—that while people may attempt to be opinion leaders due to reputation concerns, this may be a signal of weakness in some contexts, at least to those who are well informed. Understanding other factors that may drive opinion leader behavior, and their effects on consumer behavior in general, warrants future research.

A Proof of Proposition 3.2

To simplify notation, let $E(\pi_1|x_1, I)$ denote P1's expectation of P2's posterior (to period 2) expectation of π_1 , conditional on P1 taking actions x_1 and any other information P1 may have, here denoted just I . We now let x_i refer to the vector of i 's actions, $x_i = (x_{i1}, x_{i2})$. Let $Pr(G|I)$ denote P1's belief that $\theta = G$ given his information set I , and note that expected utility resulting directly from consumption (i.e., non-reputation payoff) is $Pr(G) \times 1 + (1 - Pr(G)) \times -1 = 2Pr(G) - 1$, which we refer to as *consumption utility*. It is helpful to state the following lemmas before proceeding.

Lemma A.1. *If $0.5 \leq x < y \leq 1$ and $\pi_1 \sim U[x, y]$, then: 1) $E(\pi_1|s_1 = g, \theta = G) = E(\pi_1|s_1 = b, \theta = B) = \frac{\int_x^y \pi_1^2 d\pi_1}{\int_x^y \pi_1 d\pi_1} \leq (4/9)x + (5/9)y$; 2) $E(\pi_1|s_1 = b, \theta = G) = E(\pi_1|s_1 = g, \theta = B) = \frac{\int_x^y \pi_1(1-\pi_1)d\pi_1}{\int_x^y (1-\pi_1)d\pi_1} \geq (2/3)x + (1/3)y$.*

Lemma A.2. *If $0.5 \leq x < y < z \leq 1$, then $\max\left\{\frac{\int_x^y \pi_1 d\pi_1 + \int_y^z \pi_1(1-\pi_1)d\pi_1}{\int_x^y d\pi_1 + \int_y^z (1-\pi_1)d\pi_1}, \frac{\int_x^y \pi_1 d\pi_1 + \int_y^z \pi_1^2 d\pi_1}{\int_x^y d\pi_1 + \int_y^z \pi_1 d\pi_1}\right\} \leq (4/9)x + (5/9)z$.*

The proofs are fairly straightforward and omitted. The results provide bounds on P2's posterior expectation of π_1 . The first lemma provides bounds for situations in which P1 is known to follow his signal for $\hat{\pi}_1$ in the interval $[x, y]$ (for part 1 the signal is “correct”, for

¹²We are referring to the “emperor's new clothes” phenomenon—that people may be hesitant to say a product (endorsed by opinion leaders) is bad even when they thought it was, for fear of gaining losing reputation due to evaluating the product incorrectly. See ? for an interesting sociology study on this topic.

part 2 it is “incorrect”); the second lemma builds off the first, providing bounds for situations in which P1 pools on the same action when ignoring his signal if $\hat{\pi}_1 \in [x, y]$ and following his signal if $\hat{\pi}_1 \in [y, z]$.¹³

The following is the proof of the proposition, which is done by backward induction: we first show P1’s behavior in $t = 2$ as described in the proposition is optimal (given P2’s equilibrium behavior). We then show P1’s behavior in $t = 1$ is optimal, given his behavior in $t = 2$. We then do the same for P2.

Proof. First consider P1’s decision in $t = 2$, given $x_{11} = 0$. I_{12} will include one of the four permutations of the variables x_{21} and s_1 . Suppose $x_{21} = 1$ and $s_1 = b$. P1 can infer $s_2 = g$. It is sufficient to show that, given any $\gamma_1^* \in [\underline{\pi}, \pi]$, there is a $\gamma_2^* \in (\underline{\pi}, \pi)$ such that,

$$\begin{aligned} \alpha E(\pi_1 | x_{12} = 0, x_{21} = 1, s_1 = b) &> (=)(<) 2Pr(G | x_{21} = 1, s_1 = b) - 1 + \\ &\alpha E(\pi_1 | x_{12} = 1, x_{21} = 1, s_1 = b) \end{aligned} \quad (2)$$

if and only if $\hat{\pi}_1 > (=)(<)\gamma_2^*$, with the expectations taking into account the thresholds γ_1^* and γ_2^* . That is, there exists a commonly known threshold in which P1 gets a higher expected payoff from not consuming rather than consuming in $t = 2$ only if his signal accuracy is higher than the threshold. We will also show that this threshold is unique (given γ_1^*).

Let γ_2 be the non-equilibrium analog of γ_2^* (that is, suppose there is common knowledge P2 chooses $x_{12} = 0$ iff $\hat{\pi}_1 > \gamma_2$), and let $\phi(\gamma_1^*, \gamma_2, \hat{\pi}_1)$ denote the left-hand side (LHS) of (2) minus the right-hand side (RHS). $2Pr(G | x_{21} = 1, s_1 = b) - 1$ is P1’s consumption utility, which equals $\frac{(1-\hat{\pi}_1)\pi - \hat{\pi}_1(1-\pi)}{(1-\hat{\pi}_1)\pi + \hat{\pi}_1(1-\pi)}$. This expression is strictly decreasing in $\hat{\pi}_1$ and equal to zero when $\hat{\pi}_1 = \pi$, so it is strictly negative when $\hat{\pi}_1 = \underline{\pi}$. Thus, when $\hat{\pi}_1 = \underline{\pi}$, if α is sufficiently small, then $\phi(\gamma_1^*, \gamma_2, \hat{\pi}_1) < 0$ for all $\gamma_1^*, \gamma_2 \in [\underline{\pi}, \pi]$, since the expectation terms are bounded.

It can be shown that P1’s reputation is always higher when he does not consume in $t = 2$: $E(\pi_1 | x_{12} = 0, x_{21} = 1, s_1 = b) > E(\pi_1 | x_{12} = 1, x_{21} = 1, s_1 = b)$ for all $\gamma_1^*, \gamma_2 \in [\underline{\pi}, \pi]$ and

¹³A sketch of the proof of Lemma A.2 is as follows. Both terms in the $\max\{\}$ expression are first decreasing in y , then increasing (for $y \in (x, z)$). Thus an upper bound for the expression is attained when $y = x$ or $y = z$. It can be shown the expression is highest when $y = x$, and the bound for the expression in this case is implied by Lemma A.1.

$\hat{\pi}_1 \geq \underline{\pi}$.¹⁴ Consequently when $\hat{\pi}_1 \geq \pi$, if $\alpha > 0$ then $\phi(\cdot) > 0$.

Now let $\phi^*(\gamma_1^*, \gamma_2) = \phi(\gamma_1^*, \gamma_2, \gamma_2)$; $\phi^*(\cdot)$ is just $\phi(\cdot)$ with $\hat{\pi}_1$ set equal to γ_2 . The above arguments still apply: $\phi^*(\gamma_1^*, \gamma_2)|_{\gamma_2=\pi} > 0$ and $\phi^*(\gamma_1^*, \gamma_2)|_{\gamma_2=\underline{\pi}} < 0$, for sufficiently small α . $\phi^*(\cdot)$ is continuous in its arguments so by the Intermediate Value Theorem, there exists $\gamma_2^* \in (\underline{\pi}, \pi)$ such that $\phi^*(\gamma_1^*, \gamma_2^*) = 0$. Moreover, if α is sufficiently low, then $\frac{\partial \phi^*(\cdot)}{\partial \gamma_2} > 0$ and $\frac{\partial \phi(\cdot)}{\partial \hat{\pi}} > 0$, since the partial of $2Pr(G|x_{21} = 1, s_1 = b) - 1$ with respect to $\hat{\pi}_1$ is negative and bounded below zero and the expectation terms' partial derivatives with respect to γ_2 and $\hat{\pi}_1$ are bounded.¹⁵ This implies a sufficient result for what we wanted to show: for all γ_1^* , there exists a unique $\gamma_2^* \in (\underline{\pi}, \pi)$, such that the LHS of (2) equals the RHS if and only if $\hat{\pi}_1 = \gamma_2^*$ (uniqueness follows from $\frac{\partial \phi^*(\cdot)}{\partial \gamma_2} > 0$) and the LHS of (2) is strictly greater (less) than the RHS iff $\hat{\pi}_1 > (<) \gamma_2^*$ (this follows from $\frac{\partial \phi(\cdot)}{\partial \hat{\pi}_1} > 0$).

Now suppose $x_{21} = 1$ and $s_1 = g$. Consumption utility is positive when $\hat{\pi}_1 = \underline{\pi}$, so for α sufficiently low, $\phi(\cdot) < 0$ when $\hat{\pi}_1 = \underline{\pi}$ and $s_1 = g$. And in this case it can be shown $\frac{\partial \phi(\cdot)}{\partial \hat{\pi}_1} < 0$ for low α ; the argument is analogous to the argument for $\frac{\partial \phi(\cdot)}{\partial \hat{\pi}_1} > 0$ if $s_1 = b$. Thus, it is optimal for P1 to have the strategy $x_{12}^*(s_1 = g, x_{11} = 0, x_{21} = 1) = 1$, for all $\hat{\pi}_1$.

Now suppose $x_{21} = 0$. In equilibrium, $x_{12}^*(x_{11} = 0, x_{21} = 0) = 0$, so the following incentive compatibility constraint must be satisfied (recall that ρ is the off-equilibrium-path $E(\pi_1)$):

$$\alpha E(\pi_1|x_1 = 0, x_2 = 0, s_1) > 2Pr(G|x_2 = 0, s_1) - 1 + \alpha\rho. \quad (3)$$

There are two sub-cases: 1) $s_1 = b$, 2) $s_1 = g$. In sub-case 1, $2Pr(G|I) - 1$ is bounded below zero. Thus the condition will hold for sufficiently low α , since $E(\pi_1|x_1 = 0, x_2 = 0, s_1)$ is bounded. We address sub-case 2 later in the proof.

Now consider P1's decisions in $t = 1$. γ_2^* is exogenous from the perspective of P1 when

¹⁴The LHS is clearly strictly greater than γ_2 , since that is the minimum value of $\hat{\pi}_1$ for which P1 will choose $x_{12} = 0$ in equilibrium after observing P2 consume in $t = 1$ and $\gamma_2 \leq \pi < \bar{\pi}$, and if $\gamma_2 \geq \gamma_1^*$, the RHS is weakly less than γ_2 , which is sufficient. If $\gamma_2 < \gamma_1^*$, then we use the facts that Lemma A.1 implies the LHS is weakly greater than $(2/3)\gamma_2 + (1/3)\bar{\pi}$ and Lemma A.2 implies the RHS is weakly less than $(4/9)\underline{\pi} + (5/9)\gamma_1^*$, which together can be shown to imply the statement is true, given that $\gamma_1^* \leq \pi = 0.5(\bar{\pi} + \underline{\pi})$.

¹⁵Since the derivatives are continuous on the closed interval $[\underline{\pi}, \pi]$, they must be bounded; this is easily verified. The signs of the expectation terms' partial derivatives are very likely consistent with the sign of the $\phi(\cdot)$ function's partials, but this is difficult to show. It is sufficient for the proof to exploit the fact that all the expectation terms are multiplied by α , and so for sufficiently low α the derivative of the consumption utility term is "dominant".

making his decision in $t = 1$. It is sufficient then to confirm that there exists $\gamma_1^* \in (\underline{\pi}, \pi)$ such that for all $\gamma_2^* \in [\underline{\pi}, \pi]$, if $s_1 = g$, then:

$$\begin{aligned} & 2Pr(G|s_1 = g) - 1 + \alpha E(\pi_1|x_{11} = 1, s_1 = g) \geq (<) \\ & Pr(s_2 = g|s_1 = g)(2Pr(G|s_1 = g, s_2 = g) - 1 + \alpha E(\pi_1|x_{12} = 1, x_{21} = 1)) + \\ & (1 - Pr(s_2 = g|s_1 = g)) \max\{\alpha E(\pi_1|x_1 = 0, x_2 = 0), 2Pr(G|s_1 = g, s_2 = b) - 1 + \alpha\rho\} \end{aligned} \quad (4)$$

if and only if $\hat{\pi}_1 \geq (<)\gamma_1^*$, with the expectations incorporating γ_1^*, γ_2^* . The LHS is P1's expected payoff from consuming in $t = 1$. The RHS is the expected payoff from not consuming in $t = 1$. The first term of the RHS uses the fact shown above that if $x_{21} = 1$ then (for small enough α) P1 will optimally choose $x_{12} = 1$, given $s_1 = g$, for all $\hat{\pi}_1$. The second term (the max term) allows for the fact that it may be optimal for P1 to consume in $t = 2$ even if $x_{11} = x_{21} = 0$, although this does not occur in equilibrium.

When $\hat{\pi}_1 = \underline{\pi}$, there is a gain in consumption utility from waiting/learning: $2Pr(G|s_1 = g) - 1 < Pr(s_2 = g|s_1 = g)(2Pr(G|s_1 = g, s_2 = g) - 1)$. Thus, if $\hat{\pi}_1 = \underline{\pi}$, for sufficiently low α the RHS of (4) is greater than the LHS, since all the expectation terms are bounded, making $x_{11} = 0$ optimal. To analyze optimal behavior for higher $\hat{\pi}_1$, we consider two cases for the max term of (4) in turn. In case one, for some $\hat{\pi}_1 \in (\underline{\pi}, \pi]$, $\alpha E(\pi_1|x_1 = 0, x_2 = 0) \leq 2Pr(G|s_1 = g, s_2 = b) - 1 + \alpha\rho$. Let $\hat{\pi}_1' > \underline{\pi}$ denote the minimum value of $\hat{\pi}$ such that this inequality holds (the inequality can be assured to be strict by α being sufficiently small). In this case P1 has no change in consumption utility from waiting until $t = 2$, since P1 consumes the good regardless of what P2 does in $t = 1$. And it can be shown that P1 does gain reputation from consuming in $t = 1$ for all $\hat{\pi}_1 \in [\underline{\pi}, \hat{\pi}_1']$: $E(\pi_1|x_{11} = 1, s_1 = g) > Pr(s_2 = g|s_1 = g)E(\pi_1|x_{12} = 1, I_2) + (1 - Pr(s_2 = g|s_1 = g))\rho$.¹⁶ So it will be optimal for P1 to consume in $t = 1$ if $\hat{\pi}_1 = \hat{\pi}_1'$. Also, $2Pr(G|s_1 = g, s_2 = b)$ is increasing in $\hat{\pi}_1$, while $E(\pi_1|x_1 = 0, x_2 = 0)$ is independent of $\hat{\pi}_1$ (since P2 will only condition her posterior on s_2 and x_1), so it must be true that $\alpha E(\pi_1|x_1 = 0, x_2 = 0) \leq 2Pr(G|s_1 = g, s_2 = b) - 1 + \alpha\rho$ for all $\hat{\pi}_1 \geq \hat{\pi}_1'$. Thus,

¹⁶The LHS can be shown to be greater than $(7/12)\gamma_1^* + (5/12)\bar{\pi}$ using Lemma A.1 and the facts that $E(\pi_1|x_{11} = 1, G) > 0.5(\gamma_1^* + \bar{\pi})$ and $Pr(G|s_1 = g) > 0.5$, given $\hat{\pi}_1 > \underline{\pi} \geq 0.5$. Lemma A.2 implies $E(\pi_1|x_{12} = 1, x_{21} = 1) \leq (4/9)\underline{\pi} + (5/9)\pi$, so the RHS in total is less than $0.5((4/9)\underline{\pi} + (5/9)\pi) + 0.5\pi$ since $\rho \leq \pi$. Together, these are sufficient.

the LHS of (4) is strictly greater than the RHS and P1 will prefer $x_{11} = 1$ for all $\hat{\pi}_1 \geq \hat{\pi}'_1$, and so if there exists a γ_1^* that satisfies (4), it must be weakly less than $\hat{\pi}'_1$. Note that this result implies that if the constraint of (3) would be violated ($\hat{\pi}_1$ sufficiently high and $s_1 = g$) this information set is never reached in equilibrium as P1 will choose $x_{11} = 1$. This addresses what was referred to as sub-case 2 above.

In the second case, $\forall \hat{\pi}_1 \in [\underline{\pi}, \pi]$, $\alpha E(\pi_1 | x_1 = 0, x_2 = 0) > 2Pr(G | s_1 = g, s_2 = b) - 1 + \alpha\rho$. It can be shown that there is always a reputation gain from being a first mover ($E(\pi_1 | x_{11} = 1, s_1 = g) > Pr(s_2 = g | s_1 = g)E(\pi_1 | x_{12} = 1, x_{21} = 1) + (1 - Pr(s_2 = g | s_1 = g))E(\pi_1 | x_1 = 0, x_2 = 0)$),¹⁷ and there is no gain in consumption utility if $\hat{\pi}_1 = \pi$, as $2Pr(G | s_1 = g) - 1 = Pr(s_2 = g | s_1 = g)(2Pr(G | s_1 = g, s_2 = g) - 1)$. Thus, if $\hat{\pi}_1 = \pi$, the LHS of (4) is greater than the RHS and P1 will prefer $x_{11} = 1$. Moreover, the LHS consumption utility minus that of the RHS, $2Pr(G | s_1 = g) - 1 - Pr(s_2 = g | s_1 = g)(2Pr(G | s_1 = g, s_2 = g) - 1)$ is increasing in $\hat{\pi}_1$ and the derivative is bounded above zero, and the derivative of the reputation terms with respect to $\hat{\pi}_1$ is bounded. (It is almost certainly true, but difficult to show, that the derivative of the LHS minus the RHS of (4), in total, is increasing in $\hat{\pi}_1$ as well.) Thus, we can make an argument similar to the one above using the $\phi()$ and $\phi^*()$ functions that for sufficiently small α and any $\gamma_2^* \in [\underline{\pi}, \pi]$, there is a unique $\gamma_1^* \in (\underline{\pi}, \pi)$ such that (4) is satisfied.¹⁸ Moreover, returning to case one, this same argument can be used to show there either exists $\gamma_1^* \in (\underline{\pi}, \hat{\pi}'_1)$ such that (4) is satisfied, or if not, $\gamma_1^* = \hat{\pi}'_1$, and (4) is still satisfied, as the LHS of (4) is larger than the RHS only if $\hat{\pi}_1 \geq \gamma_1^*$.

Thus, we have shown that for small α , for any $\gamma_2^* \in [\underline{\pi}, \pi]$, there is a unique $\gamma_1^* \in (\underline{\pi}, \pi)$ such that (4) holds, and that for any $\gamma_1^* \in [\underline{\pi}, \pi]$, there is a unique $\gamma_2^* \in (\underline{\pi}, \pi)$ such that (2) holds. We now make a fixed point argument to show there exists a pair, (γ_1^*, γ_2^*) , such that both (4) and (2) hold. Define a function $g_2 : [\underline{\pi}, \pi] \rightarrow (\underline{\pi}, \pi)$ by $g_2(\gamma_1^*) = \gamma_2^*$. We need to show that this function is continuous on $[\underline{\pi}, \pi]$. While it may be possible to use the Inverse

¹⁷As noted in the previous footnote, the LHS is greater than $(7/12)\gamma_1^* + (5/12)\bar{\pi}$ and $E(\pi_1 | x_{12} = 1, x_{21} = 1)$ is less than $(4/9)\underline{\pi} + (5/9)\bar{\pi}$. $E(\pi_1 | x_1 = 0, x_2 = 0)$ is the tricky term. We can again use Lemma A.2, however, to show it has an upper bound, $(4/9)\underline{\pi} + (5/9)\bar{\pi}$, since $E(\pi_1 | x_1 = 0, x_2 = 0)$ is first decreasing then increasing in γ_1^* , and is greater when $\gamma_1^* = \underline{\pi}$ and $\pi = 1$ than when $\gamma_1^* = \bar{\pi}$. This can be used to show the result, using the facts that $Pr(s_2 = g | s_1 = g) > 0.5$ and $\gamma_2^* < 0.5(\underline{\pi} + \bar{\pi})$.

¹⁸The arguments made in the previous two footnotes that refer to γ_1^* also apply to a non-equilibrium analog $\gamma_1 \in (\underline{\pi}, \pi)$, which is necessary for the $\phi()/\phi^*()$ argument above to be used.

/ Implicit Function Theorems for this purpose, as commonly stated these are for open sets, and thus do not allow us to account for behavior on the boundary.

Lemma A.3. *The function g_2 is continuous on the closed interval $[\underline{\pi}, \pi]$.*

Proof. For each positive integer n let $\delta_n = 1/n$. Suppose g_2 is not continuous at some $x_0 \in [\underline{\pi}, \pi]$. Thus there is an $\epsilon > 0$ such that for all positive integers n there is an $x_n \in [\underline{\pi}, \pi]$ such that $|x_n - x_0| < \delta_n$ but $|g_2(x_n) - g_2(x_0)| \geq \epsilon$.

As shown above, for each x_n there is a **unique** $y_n \in [\underline{\pi}, \pi]$ such that $\phi^*(x_n, y_n) = 0$; thus $y_n = g_2(x_n)$, and by assumption $|y_n - g_2(x_0)| \geq \epsilon$. As $\delta_n \rightarrow 0$, $x_n \rightarrow x_0$. Consider the sequence $\{y_n\}_{n=1}^\infty$. As each $y_n \in [\underline{\pi}, \pi]$ (a compact set), it must have a subsequence that converges, say $y_{n_k} \rightarrow y_0$; as each y_{n_k} is at least ϵ from $g_2(x_0)$, so too is y_0 (i.e., $|y_0 - g_2(x_0)| \geq \epsilon$). As ϕ^* is continuous,

$$\lim_{k \rightarrow \infty} \phi^*(x_{n_k}, y_{n_k}) = \phi^*(x_0, y_0);$$

however, $\phi^*(x_{n_k}, y_{n_k}) = 0$ for all k , thus $\phi^*(x_0, y_0) = 0$. This is a contradiction, as we know for any x there is a unique y such that $\phi^*(x, y) = 0$; for the point x_0 , we now have $\phi^*(x_0, g_2(x_0)) = 0$ and $\phi^*(x_0, y_0) = 0$ with $y_0 \neq g_2(x_0)$. Thus the function g_2 is continuous in the closed interval $[\underline{\pi}, \pi]$.

□

An analogous argument can be made for the existence of a continuous function $g_1 : [\underline{\pi}, \pi] \rightarrow (\underline{\pi}, \pi)$ such that $g_1(\gamma_2^*) = \gamma_1^*$. Thus, $g_1(g_2(x))$ is a continuous function of x , and $g_1(g_2(\underline{\pi})) - \underline{\pi} > 0$ and $g_1(g_2(\pi)) - \pi < 0$, so there exists a $\gamma_1^* \in (\underline{\pi}, \pi)$ such that $g_1(g_2(\gamma_1^*)) = \gamma_1^*$. If $\gamma_2^* = g_2(\gamma_1^*)$, then both (4) and (2) hold given the threshold pair (γ_1^*, γ_2^*) .

Now consider P1's decision in $t = 1$ if $s_1 = b$. If $\hat{\pi}_1 = \underline{\pi}$, (4) is unchanged, so $x_{11} = 0$ must still be optimal. And increasing $\hat{\pi}_1$ causes the consumption utility terms in (4) to change in the opposite direction (as compared to the case of $s_1 = g$), so the RHS of (4) must be greater than the LHS for all $\hat{\pi}_1$ for sufficiently low α , implying $x_{11}^*(b) = 0$ is optimal for all $\hat{\pi}_1$.

Finally, we check the strategy of P2. It is similar to the case in which $\alpha = 0$. In $t = 2$, it is optimal to play $x_{22} = 1$ if $x_{11} = 1 - x_{21} = 1$ and $s_2 = b$, since P2 infers P1 has a more informative signal (since $\hat{\pi}_1 \geq \gamma_1^* > \underline{\pi}$) and $s_1 = g$. And if $s_2 = g$ and $x_{11} = 0$, it is easily

shown it is optimal to play $x_{21} = 1$, from the reasoning provided in the proof of Lemma 3.1, so P2 has no incentive to wait if $s_2 = g$.

□

B Signal sequences that lead to information cascades

The following presents the various sequences of signals that lead to the formation of herding on consumption and on non-consumption. The sequences are presented for the different cases of P1's signal accuracy and realization, so the first element of each sequence (after s_1) is P2's signal, the second element (for sequences with two or more elements) is the signal of the first player in the BHW model, the third is the signal of the second BHW player, etc. For example, the sequence g implies a herd starts when P2 obtains the signal g , the sequence bg implies a herd starts when P2 has signal b and the first player in the BHW model has signal g , etc. The corresponding probabilities are fairly straightforward to calculate and the details are omitted.

1. Herding on consumption (probabilities calculated conditional on $\theta = B$)
 - a) $\hat{\pi}_1 \geq \gamma_1^*, s_1 = g$: $g, bg, bbgg, bbgbgg, bbgbgbgg, \dots$
 - b) $\hat{\pi}_1 \geq \gamma_2^*, s_1 = b$: $ggg, ggbgg, ggbgbgg, \dots$
 - c) $\hat{\pi}_1 < \gamma_1^*, s_1 = g$ or $\hat{\pi}_1 \leq \gamma_2^*, s_1 = b$: $gg, gbgg, ggbgg, \dots$
2. Herding on non-consumption (probabilities calculated conditional on $\theta = G$)
 - a) $\hat{\pi}_1 \geq \gamma_1^*, s_1 = g$: $bbb, bbgbb, bbgbgbb, \dots$
 - b) $\hat{\pi}_1 \geq \gamma_2^*, s_1 = b$: $b, gb, ggbb, ggbgbb, \dots$
 - c) $\hat{\pi}_1 < \gamma_1^*, s_1 = g$ or $\hat{\pi}_1 \leq \gamma_2^*, s_1 = b$: $b, gbb, gbgbb, \dots$

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