Introduction 000000	General Theory 0000000	Applications	3 <i>x</i> + 1 0000	Copulas 0000	Conclusions

# Benford's law, or: Why the IRS cares about number theory!

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Smith College, November 15, 2011

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Introductio	on				

For a nice data set, such as the Fibonacci numbers, stock prices, street addresses of Smith professors, ..., what percent of the leading digits are 1?

Plausible answers:



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Introductio	on				

For a nice data set, such as the Fibonacci numbers, stock prices, street addresses of Smith professors, ..., what percent of the leading digits are 1?

Plausible answers: 10%



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Introduction	on				

For a nice data set, such as the Fibonacci numbers, stock prices, street addresses of Smith professors, ..., what percent of the leading digits are 1?

Plausible answers: 10%, 11%

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Introductio	on				

For a nice data set, such as the Fibonacci numbers, stock prices, street addresses of Smith professors, ..., what percent of the leading digits are 1?

Plausible answers: 10%, 11%, about 30%.



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Summary					

- State Benford's Law.
- Discuss examples and applications.
- Sketch proofs.
- Describe open problems.

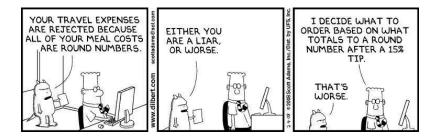


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Caveats!					

• A math test indicating fraud is *not* proof of fraud: unlikely events, alternate reasons.

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#### Statement

For many data sets, probability of observing a first digit of *d* base *B* is  $\log_B(\frac{d+1}{d})$ ; base 10 about 30% are 1s.

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### • Not all data sets satisfy Benford's Law.

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 ◊ Long street [1, *L*]: *L* = 199 versus *L* = 999.

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  - $\diamond$  Oscillates between 1/9 and 5/9 with first digit 1.

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  - $\diamond$  Long street [1, *L*]: *L* = 199 versus *L* = 999.
  - $\diamond$  Oscillates between 1/9 and 5/9 with first digit 1.
  - Many streets of different sizes: close to Benford.

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Examples					

- recurrence relations
- special functions (such as n!)
- iterates of power, exponential, rational maps
- products of random variables
- L-functions, characteristic polynomials
- iterates of the 3x + 1 map
- differences of order statistics
- hydrology and financial data
- many hierarchical Bayesian models

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Applicatio	ons				

# analyzing round-off errors

- determining the optimal way to store numbers
- detecting tax and image fraud, and data integrity

Introduction	General Theory	Applications	Copulas	Conclusions

# **General Theory**

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Mantissa	s (or Significar	nds)			

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Mantissa	s (or Significan	nds)			

Mantissa:  $x = M_{10}(x) \cdot 10^k$ , *k* integer.

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Mantissa:  $x = M_{10}(x) \cdot 10^k$ , k integer.

 $M_{10}(x) = M_{10}(\tilde{x})$  if and only if x and  $\tilde{x}$  have the same leading digits.

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Key observation:  $\log_{10}(x) = \log_{10}(\tilde{x}) \mod 1$  if and only if x and  $\tilde{x}$  have the same leading digits. Thus often study  $y = \log_{10} x$ .

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Fauidistr	ibution and Be	nford's I aw			

 $\{y_n\}_{n=1}^{\infty}$  is equidistributed modulo 1 if probability  $y_n \mod 1 \in [a, b]$  tends to b - a:

$$\frac{\#\{n \le N : y_n \bmod 1 \in [a, b]\}}{N} \rightarrow b - a.$$

Introduction 000000	General Theory ○●○○○○○	Applications	3 <i>x</i> + 1 0000	Copulas 0000	Conclusions
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• Thm:  $\beta \notin \mathbb{Q}$ ,  $n\beta$  is equidistributed mod 1.

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• Examples: 
$$\log_{10} 2$$
,  $\log_{10} \left(\frac{1+\sqrt{5}}{2}\right) \notin \mathbb{Q}$ .

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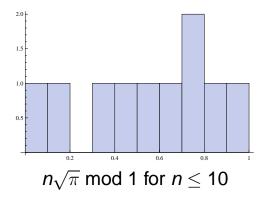
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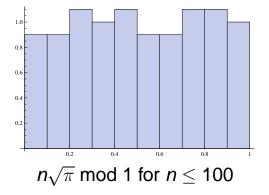
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- Examples:  $\log_{10} 2$ ,  $\log_{10} \left(\frac{1+\sqrt{5}}{2}\right) \notin \mathbb{Q}$ . *Proof:* if rational:  $2 = 10^{p/q}$ . Thus  $2^q = 10^p$  or  $2^{q-p} = 5^p$ , impossible.

Introduction	General Theory ○O●○○○○	Applications	3 <i>x</i> + 1 0000	Copulas 0000	Conclusions
Example	of Equidistribu	<b>ition:</b> $n_{\sqrt{\pi}}$ mo	nd 1		



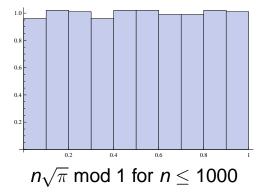
Introduction 000000	General Theory ○O●○○○○	Applications	3x + 1 0000	Copulas 0000	Conclusions

#### **Example of Equidistribution:** $n\sqrt{\pi} \mod 1$



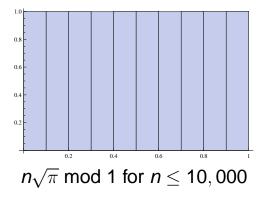
Introduction 000000	General Theory ○O●○○○○	Applications	3x + 1 0000	Copulas 0000	Conclusions

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Introduction 000000	General Theory ○O●○○○○	Applications	3x + 1 0000	Copulas 0000	Conclusions

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Logarithr	ns and Benford	l's Law			

# **Fundamental Equivalence**

Data set  $\{x_i\}$  is Benford base *B* if  $\{y_i\}$  is equidistributed mod 1, where  $y_i = \log_B x_i$ .

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## **Fundamental Equivalence**

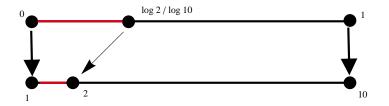
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Introduction 000000	General Theory	Applications	3 <i>x</i> + 1 0000	Copulas 0000	Conclusions
Examples					

# • $2^n$ is Benford base 10 as $\log_{10} 2 \notin \mathbb{Q}$ .

Introduction 000000	General Theory ○○○○●○○	Applications	3x + 1 0000	Copulas 0000	Conclusions
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# • Fibonacci numbers are Benford base 10.

Introduction 000000	General Theory ○○○○●○○	Applications	3x + 1 0000	Copulas 0000	Conclusions
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- $2^n$  is Benford base 10 as  $\log_{10} 2 \notin \mathbb{Q}$ .
- Fibonacci numbers are Benford base 10.  $a_{n+1} = a_n + a_{n-1}$ .

Introduction	General Theory ○○○○●○○	Applications	3x + 1 0000	Copulas 0000	Conclusions
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$$a_{n+1} = a_n + a_{n-1}.$$
  
Guess  $a_n = r^n$ :  $r^{n+1} = r^n + r^{n-1}$  or  $r^2 = r + 1.$ 

Introduction 000000	General Theory ○○○○●○○	Applications	3 <i>x</i> + 1 0000	Copulas 0000	Conclusions
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Introduction 000000	General Theory ○○○○●○○	Applications	3 <i>x</i> + 1 0000	Copulas 0000	Conclusions
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 $a_{n+1} = a_n + a_{n-1}.$ Guess  $a_n = r^n$ :  $r^{n+1} = r^n + r^{n-1}$  or  $r^2 = r + 1.$ Roots  $r = (1 \pm \sqrt{5})/2.$ General solution:  $a_n = c_1 r_1^n + c_2 r_2^n.$ 

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#### Most linear recurrence relations Benford.

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<b>Digits of</b> 2 <sup><i>r</i></sup>	)				

#### First 60 values of $2^n$ (only displaying 30)

				/		
1	1024	1048576	digit	#	Obs Prob	Benf Prob
2	2048	2097152	1	18	.300	.301
4	4096	4194304	2	12	.200	.176
8	8192	8388608	3	6	.100	.125
16	16384	16777216	4	6	.100	.097
32	32768	33554432	5	6	.100	.079
64	65536	67108864	6	4	.067	.067
128	131072	134217728	7	2	.033	.058
256	262144	268435456	8	5	.083	.051
512	524288	536870912	9	1	.017	.046

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<b>Digits of</b> 2 <sup><i>r</i></sup>	)				

#### First 60 values of $2^n$ (only displaying 30)

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16163841677721646.100.09732327683355443256.100.07964655366710886464.067.06712813107213421772872.033.05825626214426843545685.083.051	4	4096	4194304	2	12	.200	.176
32327683355443256.100.07964655366710886464.067.06712813107213421772872.033.05825626214426843545685.083.051	8	8192	8388608	3	6	.100	.125
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12813107213421772872.033.05825626214426843545685.083.051	32	32768	33554432	5	6	.100	.079
256 262144 268435456 8 5 .083 .051	64	65536	67108864	6	4	.067	.067
	128	131072	134217728	7	2	.033	.058
512 524288 536870912 9 1 .017 .046	256	262144	268435456	8	5	.083	.051
	512	524288	536870912	9	1	.017	.046

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Digits of 2 <sup>r</sup>	1				

First 60 values of  $2^n$  (only displaying 30):  $2^{10} = 1024 \approx 10^3$ .

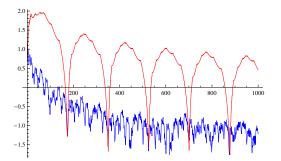
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Introduction 000000	General Theory ○○○○○●	Applications	3x + 1 0000	Copulas 0000	Conclusions
Logarithr	ns and Benford	d's Law			

$\chi^2$ va	alues fo			N (5% 1	5.5).
	N	$\chi^2(\gamma)$	$\chi^2(e)$	$\chi^2(\pi)$	
	100	0.72	0.30	46.65	
	200	0.24	0.30	8.58	
	400	0.14	0.10	10.55	
	500	0.08	0.07	2.69	
	700	0.19	0.04	0.05	
	800	0.04	0.03	6.19	
	900	0.09	0.09	1.71	
	1000	0.02	0.06	2.90	

Introduction	General Theory	Applications	3x + 1	Copulas	Conclusions
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Logarithn	ns and Benford	d's Law: Base	e 10		

 $\log_{10}(\chi^2)$  vs *N* for  $\pi^n$  (red) and  $e^n$  (blue),  $n \in \{1, ..., N\}$ . Note  $\pi^{175} \approx 1.0028 \cdot 10^{87}$ , (5% and 8 d.f.,  $\log_{10}(\chi^2) \approx .44$ ).



Introduction	General Theory	Applications	Copulas	Conclusions

# Applications

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#### **Applications for the IRS: Detecting Fraud**

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#### Applications for the IRS: Detecting Fraud

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Introduction 000000	General Theory 0000000	Applications	3x + 1 0000	Copulas 0000	Conclusions
Detecting	Fraud				

# Audit of a bank revealed huge spike of numbers

Introduction 000000	General Theory	Applications	3x + 1 0000	Copulas 0000	Conclusions
Detecting	Fraud				

# Audit of a bank revealed huge spike of numbers starting with 4

Introduction 000000	General Theory	Applications	3x + 1 0000	Copulas 0000	Conclusions
Detecting	Fraud				

# Audit of a bank revealed huge spike of numbers starting with 48 and 49

Introduction 000000	General Theory	Applications	3x + 1 0000	Copulas 0000	Conclusions
Detecting	Fraud				

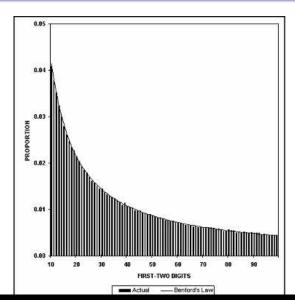
 Audit of a bank revealed huge spike of numbers starting with 48 and 49, most due to one person.

Introduction	General Theory 0000000	Applications ○●○○	3x + 1 0000	Copulas 0000	Conclusions
Detecting	Fraud				

- Audit of a bank revealed huge spike of numbers starting with 48 and 49, most due to one person.
- Write-off limit of \$5,000. Officer had friends applying for credit cards, ran up balances just under \$5,000 then he would write the debts off.

Introduction	General Theory	Applications	Copulas	Conclusions
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#### Data Integrity: Stream Flow Statistics: 130 years, 457,440 records



Introduction 000000	General Theory 0000000	Applications	3 <i>x</i> + 1 0000	Copulas 0000	Conclusions
Election I	Fraud: Iran 200	)9			

Numerous protests/complaints over Iran's 2009 elections.

Lot of analysis; data moderately suspicious:

- First and second leading digits;
- Last two digits (should almost be uniform);
- Last two digits differing by at least 2.

Warning: enough tests, even if nothing wrong will find a suspicious result (but when all tests are on the boundary...).

Introduction	General Theory 0000000	Applications	3x + 1 0000	Copulas 0000	Conclusions

The 3x + 1 Problem and Benford's Law

Introduction 000000	General Theory 0000000	Applications	3x + 1 ●○○○	Copulas 0000	Conclusions
3 <i>x</i> + 1 <b>Pr</b>	oblem				

• Kakutani (conspiracy), Erdös (not ready).

• x odd, 
$$T(x) = \frac{3x+1}{2^k}$$
,  $2^k ||3x+1$ .

• Conjecture: for some n = n(x),  $T^n(x) = 1$ .

• 7 
$$ightarrow_1$$
 11  $ightarrow_1$  17  $ightarrow_2$  13  $ightarrow_3$  5  $ightarrow_4$  1  $ightarrow_2$  1

Introduction 000000	General Theory 0000000	Applications	3x + 1 $\circ \bullet \circ \circ$	Copulas 0000	Conclusions
3 <i>x</i> + 1 an	d Benford				

#### Theorem (Kontorovich and M–, 2005)

As  $m \to \infty$ ,  $x_m/(3/4)^m x_0$  is Benford.

# Theorem (Lagarias-Soundararajan 2006)

 $X \ge 2^N$ , for all but at most  $c(B)N^{-1/36}X$  initial seeds the distribution of the first N iterates of the 3x + 1 map are within  $2N^{-1/36}$  of the Benford probabilities.



Introduction 000000	General Theory 0000000	Applications	3x + 1	Copulas 0000	Conclusions
Sketch of	the proof				

• Failed Proof: lattices, bad errors.

• CLT: 
$$(S_m - 2m)/\sqrt{2m} \to N(0, 1)$$
:

$$\mathbb{P}\left(\mathsf{S}_m-2m=k\right)=\frac{\eta(k/\sqrt{m})}{\sqrt{m}}+O\left(\frac{1}{g(m)\sqrt{m}}\right).$$

• Quantified Equidistribution:  $I_{\ell} = \{\ell M, \dots, (\ell+1)M - 1\},\$   $M = m^c, \ c < 1/2$   $k_1, k_2 \in I_{\ell}: \left| \eta\left(\frac{k_1}{\sqrt{m}}\right) - \eta\left(\frac{k_2}{\sqrt{m}}\right) \right|$  small  $C = \log_B 2$  of irrationality type  $\kappa < \infty$ :

$$\#\{k \in I_{\ell} : \overline{kC} \in [a, b]\} = M(b - a) + O(M^{1 + \epsilon - 1/\kappa}).$$

Introduction 000000	General Theory 0000000	Applications	3x + 1	Copulas 0000	Conclusions
Sketch of	the proof				

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$$\#\{k \in I_{\ell} : \overline{kC} \in [a, b]\} = M(b - a) + O(M^{1 + \epsilon - 1/\kappa}).$$

Introduction 000000	General Theory	Applications	3x + 1 $\circ \circ \circ \bullet$	Copulas 0000	Conclusions

3x + 1 Data: random 10,000 digit number,  $2^{k}||3x + 1$ 

# 80,514 iterations ( $(4/3)^n = a_0$ predicts 80,319); $\chi^2 = 13.5$ (5% 15.5).

Digit	Number	Observed	Benford
1	24251	0.301	0.301
2	14156	0.176	0.176
3	10227	0.127	0.125
4	7931	0.099	0.097
5	6359	0.079	0.079
6	5372	0.067	0.067
7	4476	0.056	0.058
8	4092	0.051	0.051
9	3650	0.045	0.046

Introduction 000000	General Theory 0000000	Applications 0000	3x + 1	Copulas 0000	Conclusions			
3x + 1 Data: random 10,000 digit number, $2 3x + 1$								

# 241,344 iterations, $\chi^2 = 11.4$ (5% 15.5).

Digit	Number	Observed	Benford
1	72924	0.302	0.301
2	42357	0.176	0.176
3	30201	0.125	0.125
4	23507	0.097	0.097
5	18928	0.078	0.079
6	16296	0.068	0.067
7	13702	0.057	0.058
8	12356	0.051	0.051
9	11073	0.046	0.046

Introduction	General Theory	Applications	3 <i>x</i> + 1	Copulas	Conclusions
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# Copulas and Benford's Law (joint with Thealexa Becker '13)

Introduction 000000	General Theory 0000000	Applications	3 <i>x</i> + 1 0000	Copulas ●○○○	Conclusions

# **Definition of Copulas**

Copula: A form of joint CDF between multiple variables with given uniform marginals on the d-dimensional unit cube.

#### **Sklar's Theorem**

Let X and Y be random variables with joint distribution function H and marginal distribution fucntions F and G respectively. There exists a copula, C, such that

for all  $x, y \in \mathbb{R}$ , H(x, y) = C(F(x), G(y)).

Introduction	General Theory	Applications	3 <i>x</i> + 1 0000	Copulas ○●○○	Conclusions

### **Archimedean Copulas**

A commonly used / studied family of copulas is of the form

$$C(\boldsymbol{x},\boldsymbol{y}) = \phi^{-1}(\phi(\boldsymbol{x}) + \phi(\boldsymbol{y}))$$

where  $\phi$  is the generator and  $\phi^{-1}$  is the inverse generator of the copula.

Introduction	General Theory 0000000	Applications	3 <i>x</i> + 1 0000	Copulas ○○●○	Conclusions

Investigating the Benfordness of the product of random variables arising from copulas.

Clayton Copula:  $C(x, y) = (x^{-\theta} + y^{-\theta} - 1)^{-1/\theta}$ .

**PDF (bivariate):**  $\theta(\theta^{-1} + 1)(xy)^{-\theta - 1}(x^{-\theta} + y^{-\theta} - 1)^{-2-1/\theta}$ .

 $\begin{array}{l} \text{PDF (general case):} \\ \theta^{n-1} \frac{\Gamma(n+\theta^{-1})}{\Gamma(1+\theta^{-1})} (x_1 \cdots x_n)^{-\theta-1} (x_1^{-\theta} + \cdots + x_n^{-\theta} - 1)^{-n-1/\theta}. \end{array}$ 

Introduction 000000	General Theory 0000000	Applications	3x + 1 0000	Copulas ○○○●	Conclusions
Results					

- Early data and chi-square tests of multivariate copulas suggest Benford behavior of the products of copulas.
- Proof strategy includes the integration of the PDF over the region in which the product has first digit *d* using Poisson summation:

$$\int_0^1 \cdots \int_0^1 \sum_k \widehat{\phi}_{\log_{10}(x_1 \cdots x_n)}(k) p(x_1, \dots, x_n) dx_1 \cdots dx_n,$$

where

$$\phi_a(u) = \chi_{[1,2)}(10^{u+a}) = \begin{cases} 1 & \text{if } 10^{u+a} \in [1,2) \\ 0 & \text{otherwise.} \end{cases}$$

Introduction	General Theory	Applications	Copulas	Conclusions

# Conclusions

Introduction	General Theory	Applications	3x + 1	Copulas	Conclusions			
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Conclusions and Future Investigations								

- Many different systems are Benford.
- Ingredients of proofs (logarithms, equidistribution).
- Applications to fraud detection / data integrity.

# • Future work:

- ◊ Study digits of other systems.
- Oevelop more sophisticated tests for fraud.