TEACHING OF STATISTICS
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Statistical Investigations as a Tool in Undergraduate Mathematics Research

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Abstract

Many widely believed conjectures have little numerical evidence. With the increasing availability of cheap computational power, many interesting cases are within the reach of undergraduates. Further, the algorithms necessary to investigate these problems are often interesting in their own right, and provide an excellent opportunity to introduce students to research problems.

This provides an ideal situation to involve undergraduates in cutting edge research. They numerically and theoretically investigate active areas of mathematics, seeing both what we do, and what it is like to do it. For the faculty, instead of teaching another standard cookbook class, they are able to lecture and guide work in their research areas.

We will discuss the new Undergraduate Mathematics Laboratory at Princeton (design and implementation, from both the faculty and student perspectives), as well as some of the results obtained.

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Introduction to Continued Fractions

Continued Fraction Expansions:

$$
\begin{aligned}
x &= a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cdots}}}, \quad a_i \text{ a positive integer} \\
&= [a_0, a_1, a_2, \ldots]
\end{aligned}
$$

Determination of the a_is:

1. $a_0 = [x]$**, the greatest integer at most** x**.**

2. $a_1 = \left[\frac{1}{x - a_0}\right]$**. Let** $x_1 = \frac{1}{x - a_0}$**.**

3. $a_2 = \left[\frac{1}{x_1 - a_0}\right]$**.**

And so on...

Properties of Digits

Rational Numbers:

x is rational if and only if the continued fraction expansion of x terminates.

Quadratic Irrationals:

x is a quadratic irrational (ie, solves $ax^2 + bx + c = 0$) if and only if the continued fraction expansion of x is periodic.

Gauss-Kuzmin Theorem:

For almost all $x \in [0, 1)$ (in the sense of Lebesgue measure),

$$\lim_{n\to\infty} \textbf{Prob}\,(a_n(x) \;=\; k) \;=\; \log_2\left(1 + \frac{1}{k(k+2)}\right).$$

Let $\mathbf{I_{GK}} \subset$ [0,1) denote all x which, in the limit, have distribution of digits satisfying the Gauss-Kuzmin Law.