Pythagoras at the Bat: An Introduction to Stats and Modeling

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Acknowledgments

Sal Baxamusa, Phil Birnbaum, Chris Chiang, Ray Ciccolella, Steve Johnston, Michelle Manes, Russ Mann, students of Math 162 and Math 197 at Brown, Math 150 and 399 at Williams.

Dedicated to my great uncle Newt Bromberg (a lifetime Red Sox fan who promised me that I would live to see a World Series Championship in Boston).

Chris Long and the San Diego Padres.

Acknowledgments

Thoughts on Research

Research: What questions to ask? How? With whom?

- Build on what you know and can learn.
- What will be interesting?
- How will you work?
- Where are the questions?

Utilize: What are your tools and how can they be used?

Law of the Hammer:

- Abraham Kaplan: I call it the law of the instrument, and it may be formulated as follows: Give a small boy a hammer, and he will find that everything he encounters needs pounding.
- Abraham Maslow: I suppose it is tempting, if the only tool you have is a hammer, to treat everything as if it were a nail.
- Bernard Baruch: If all you have is a hammer, everything looks like a nail.

Introduction to the Pythagorean Won–Loss Theorem

Goals of the Talk

- Give derivation Pythagorean Won–Loss formula.
- Observe ideas / techniques of modeling.
- See how advanced theory enters in simple problems.
- Opportunities from inefficiencies.
- Xtra: further avenues for research.

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GO SOX!

Statistics

Goal is to find good statistics to describe real world.

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Figure: Harvard Bridge, about 620.1 meters.

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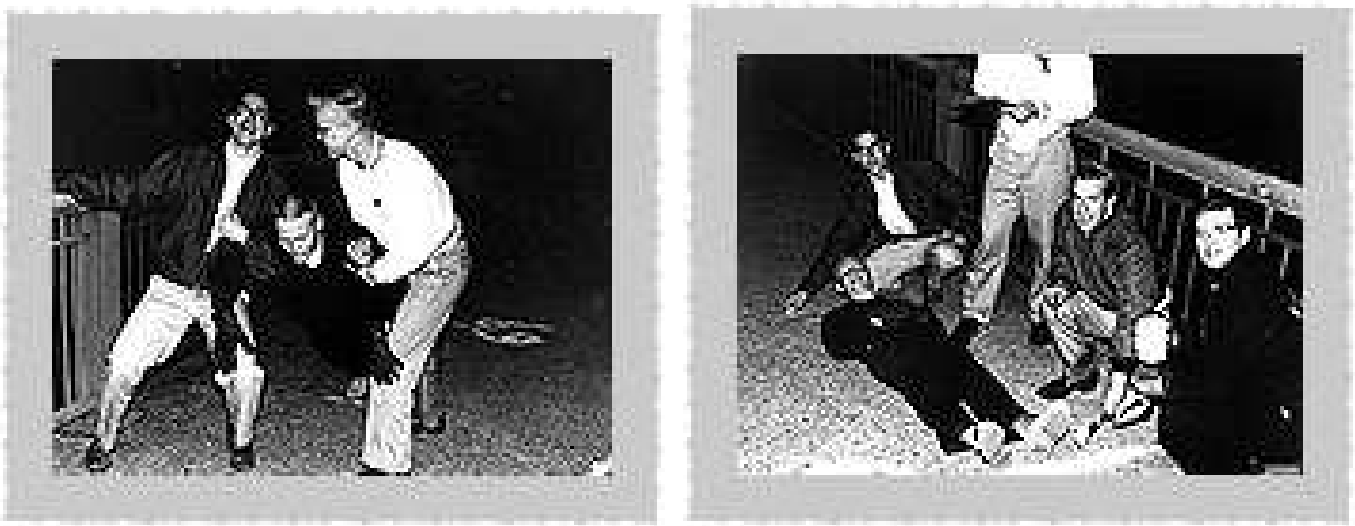

Figure: Harvard Bridge, 364.1 Smoots ($\pm$ one ear).

Baseball Review

Goal:

Numerical Observation: Pythagorean Won–Loss Formula

Parameters

- $\mathrm{RS}_{\mathrm{obs}}$: average number of runs scored per game;
- $\mathrm{RA}_{\mathrm{obs}}$: average number of runs allowed per game;
- γ: some parameter, constant for a sport.

Numerical Observation: Pythagorean Won–Loss Formula

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- γ: some parameter, constant for a sport.

James' Won–Loss Formula (NUMERICAL Observation)

$$\mathrm{Won-Loss\ Percentage} = \frac{\#\mathrm{Wins}}{\#\mathrm{Games}} = \frac{\mathrm{RS}_{\mathrm{obs}}^{\gamma}}{\mathrm{RS}_{\mathrm{obs}}^{\gamma} + \mathrm{RA}_{\mathrm{obs}}^{\gamma}}$$

γ originally taken as 2, numerical studies show best γ for baseball is about 1.82.

Pythagorean Won–Loss Formula: Example

James' Won–Loss Formula

$$\text{Won} - \text{Loss Percentage} = \frac{\#\text{Wins}}{\#\text{Games}} = \frac{\text{RS}_{\text{obs}}^{\gamma}}{\text{RS}_{\text{obs}}^{\gamma} + \text{RA}_{\text{obs}}^{\gamma}}$$

Example ($\gamma = 1.82$): In 2009 the Red Sox were 95–67. They scored 872 runs and allowed 736, for a Pythagorean prediction record of 93.4 wins and 68.6 losses; the Yankees were 103–59 but predicted to be 95.2–66.8 (they scored 915 runs and allowed 753).

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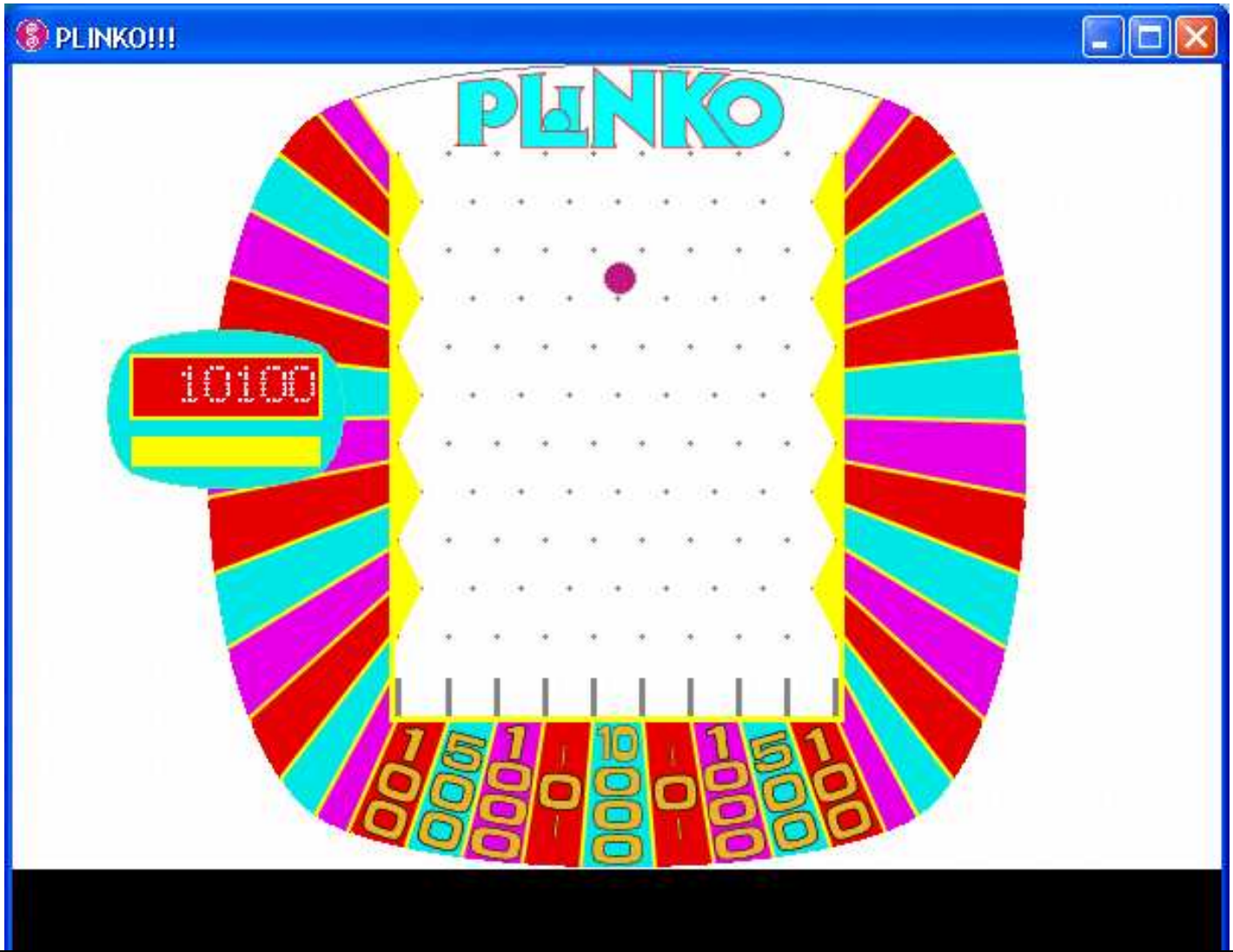
2011: Red Sox 'should' be 95-67, Tampa 'should' be 92-70....

Applications of the Pythagorean Won–Loss Formula

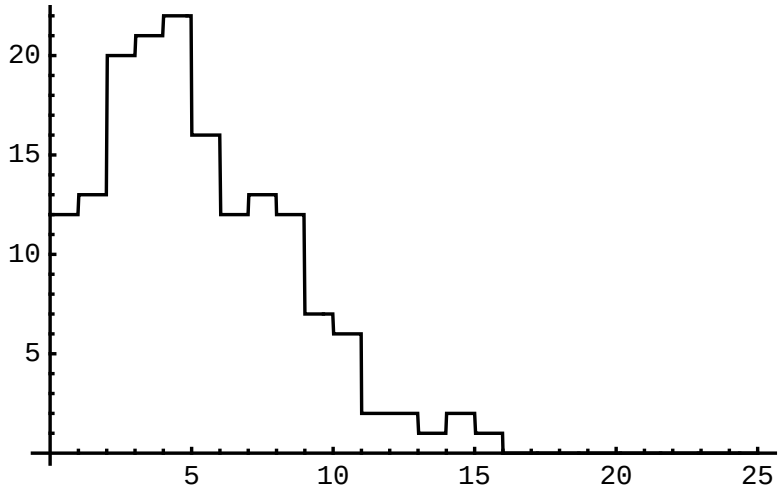
- Extrapolation: use half-way through season to predict a team's performance for rest of season.
- Evaluation: see if consistently over-perform or under-perform.
- Advantage: Other statistics / formulas (run-differential per game); this is easy to use, depends only on two simple numbers for a team.

Probability and Modeling

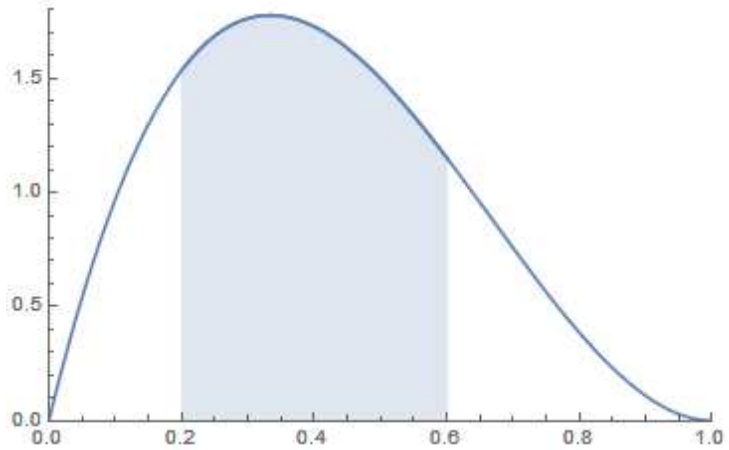


Observed scoring distributions

Goal is to model observed scoring distributions; for example, consider

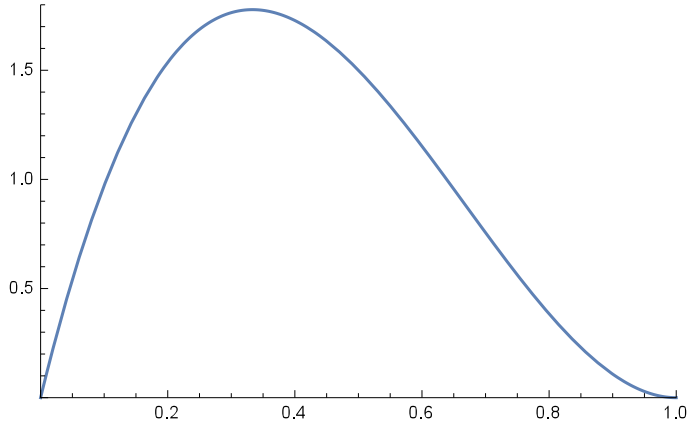


Probability Review



- Let X be random variable with density $p(x)$:
  - $p(x) \geq 0$;
  - $\int_{-\infty}^{\infty} p(x)\mathrm{d}x = 1$;
  - $\mathrm{Prob}\,(a \leq X \leq b) = \int_a^b p(x)\mathrm{d}x$.

Probability Review



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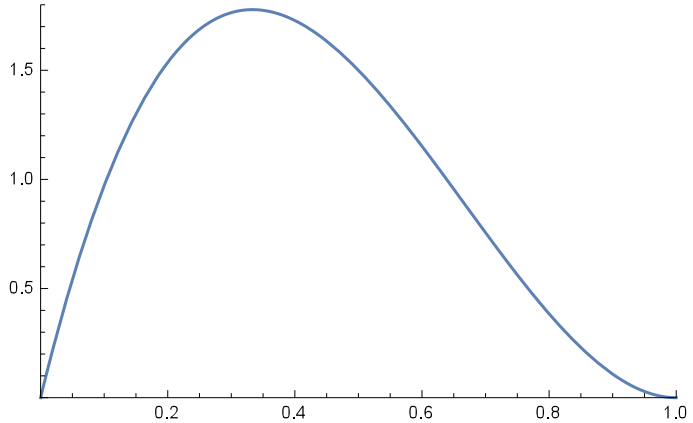
  - $\mathrm{Prob}\,(a \leq X \leq b) = \int_a^b p(x)\mathrm{d}x$.
- Mean $\mu = \int_{-\infty}^{\infty} xp(x)\mathrm{d}x$.

Probability Review



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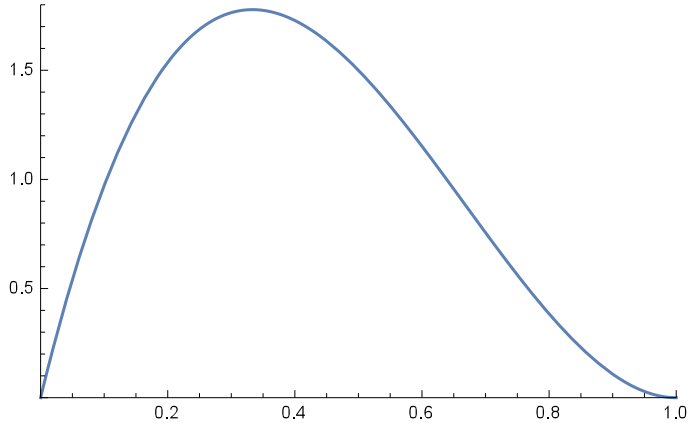
- Mean $\mu = \int_{-\infty}^{\infty} xp(x)\mathrm{d}x$.
- Variance $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\mathrm{d}x$.

Probability Review



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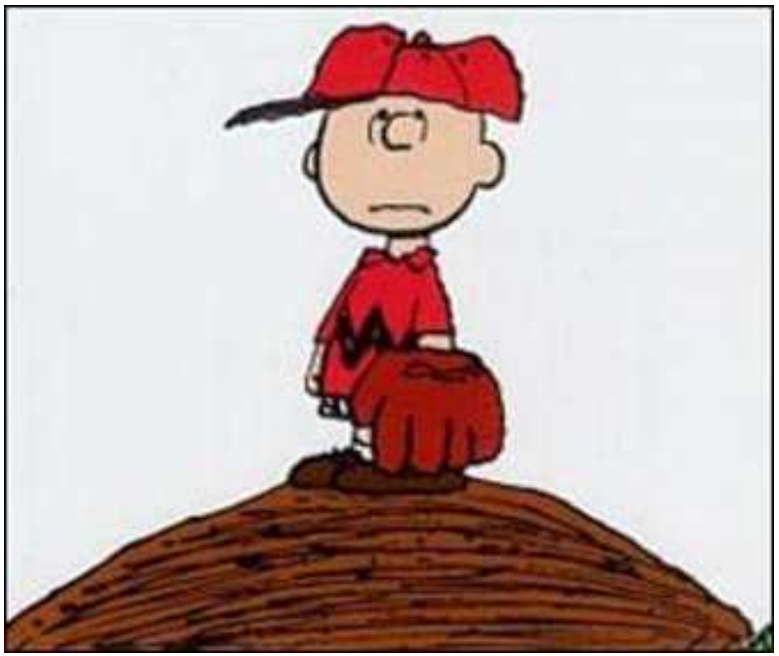
  - $\mathrm{Prob}\,(a \leq X \leq b) = \int_a^b p(x)\mathrm{d}x$.
- Mean $\mu = \int_{-\infty}^{\infty} xp(x)\mathrm{d}x$.
- Variance $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\mathrm{d}x$.
- Independence: knowledge of one random variable gives no knowledge of the other.

Modeling the Real World

Guidelines for Modeling:

- Model should capture key features of the system;
- Model should be mathematically tractable (solvable).



Modeling the Real World (cont)

Possible Model:

- Runs Scored and Runs Allowed independent random variables;
- $f_{\mathrm{RS}}(x)$, $g_{\mathrm{RA}}(y)$: probability density functions for runs scored (allowed).

Modeling the Real World (cont)

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- Runs Scored and Runs Allowed independent random variables;
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Won–Loss formula follows from computing

$$\int_{x=0}^{\infty} \left[\int_{y \le x} f_{\mathrm{RS}}(x) g_{\mathrm{RA}}(y) \mathrm{d}y \right] \mathrm{d}x \quad \text{or} \quad \sum_{i=0}^{\infty} \left[\sum_{j<i} f_{\mathrm{RS}}(i) g_{\mathrm{RA}}(j) \right].$$

Problems with the Model

Reduced to calculating

$$\int_{x=0}^{\infty} \left[\int_{y \leq x} f_{\mathrm{RS}}(x) g_{\mathrm{RA}}(y) \mathrm{d}y \right] \mathrm{d}x \quad \text{or} \quad \sum_{i=0}^{\infty} \left[\sum_{j<i} f_{\mathrm{RS}}(i) g_{\mathrm{RA}}(j) \right].$$

Problems with the Model

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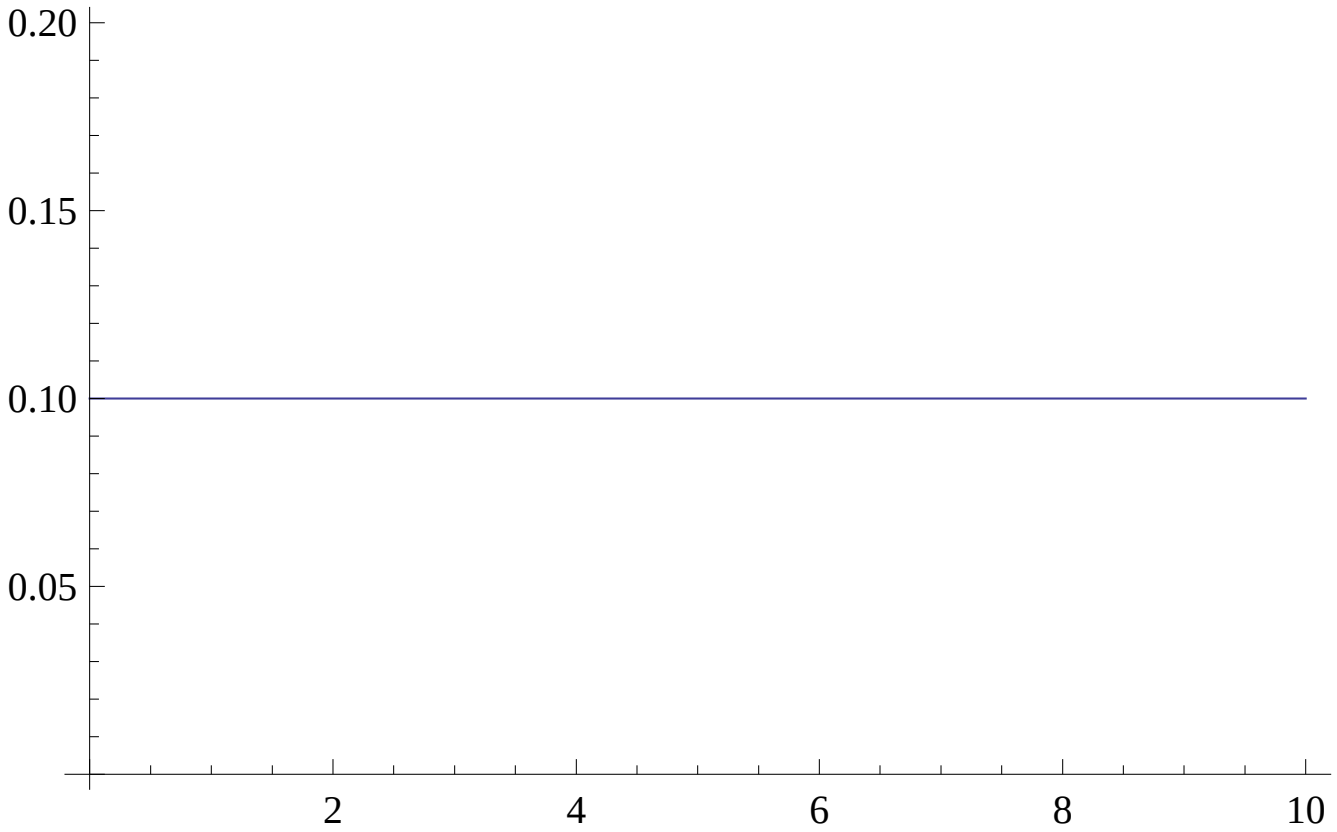
Problems with the model:

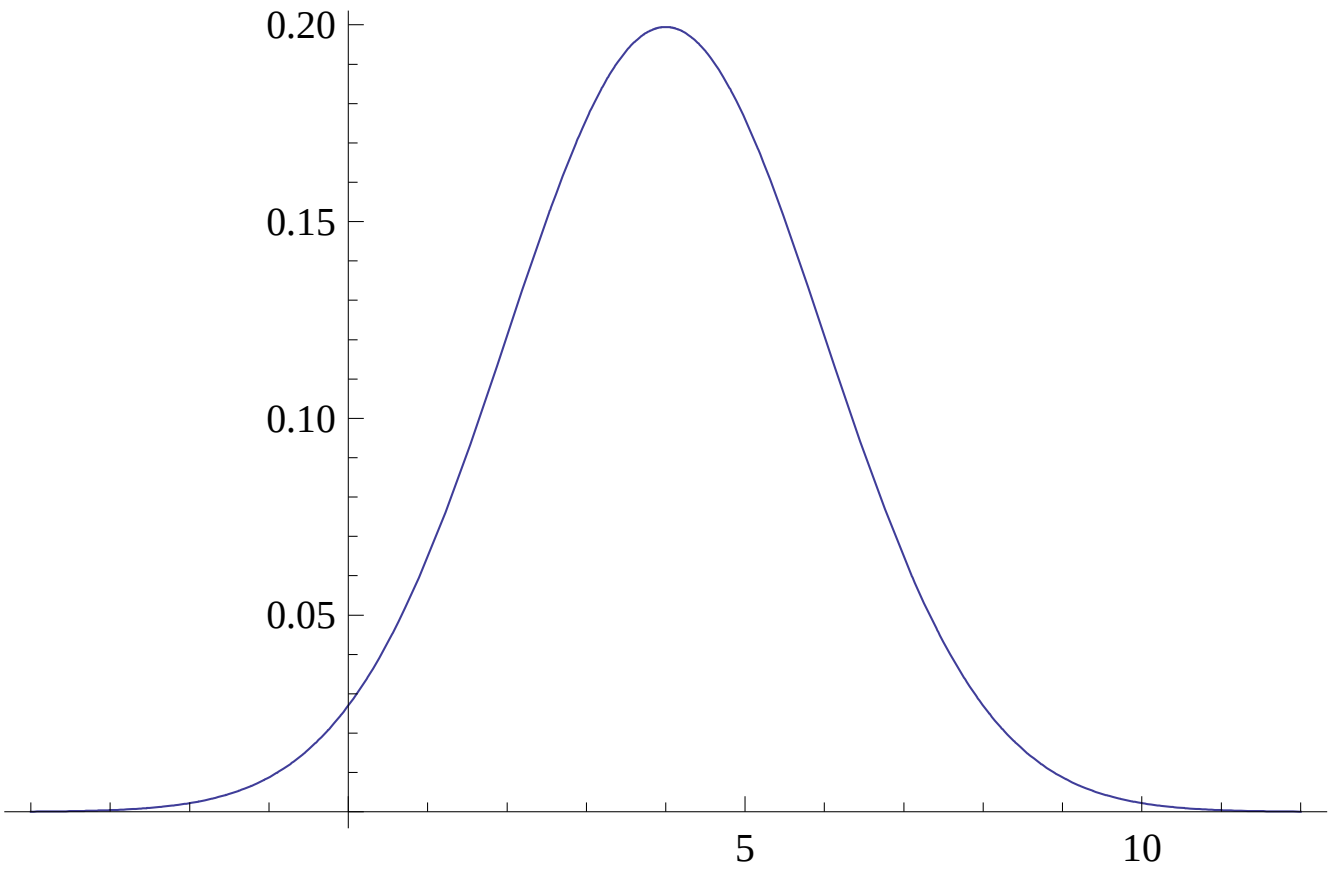
- What are explicit formulas for f_{RS} and g_{RA}?
- Are the runs scored and allowed independent random variables?
- Can the integral (or sum) be computed in closed form?

Choices for f_{RS} and g_{RA}



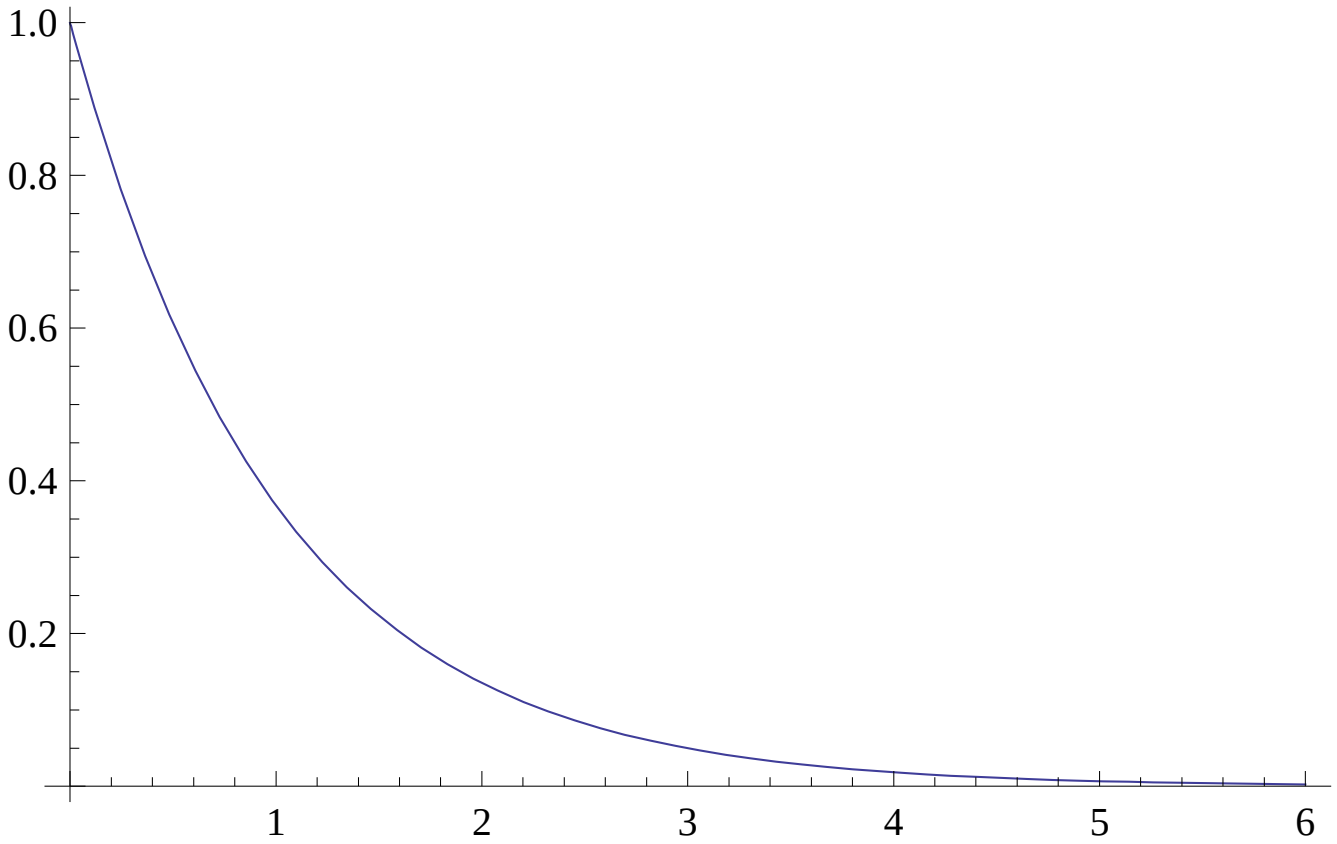
Uniform Distribution on $[0, 10]$.

Choices for f_{RS} and g_{RA}



Normal Distribution: mean 4, standard deviation 2.

Choices for f_{RS} and g_{RA}



Exponential Distribution: e^{-x}.

Three Parameter Weibull

Weibull distribution:

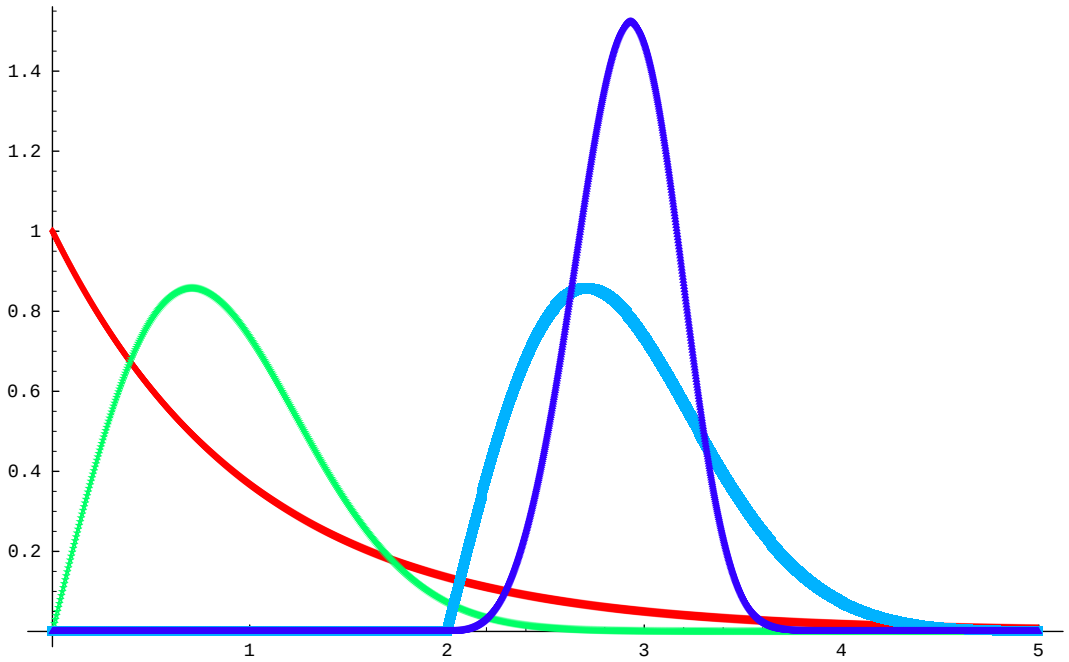
$$f(x;\alpha,\beta,\gamma) = \begin{cases} \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1} e^{-((x-\beta)/\alpha)^{\gamma}} & \text{if } x \geq \beta \\ 0 & \text{otherwise.} \end{cases}$$

- α: scale (variance: meters versus centimeters);
- β: origin (mean: translation, zero point);
- γ: shape (behavior near β and at infinity).

Various values give different shapes, but can we find α, β, γ such that it fits observed data? Is the Weibull justifiable by some reasonable hypotheses?

Weibull Plots: Parameters (α, β, γ):

$$f(x;\alpha,\beta,\gamma) = \begin{cases} \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1} e^{-((x-\beta)/\alpha)^{\gamma}} & \text{if } x \geq \beta \\ 0 & \text{otherwise.} \end{cases}$$



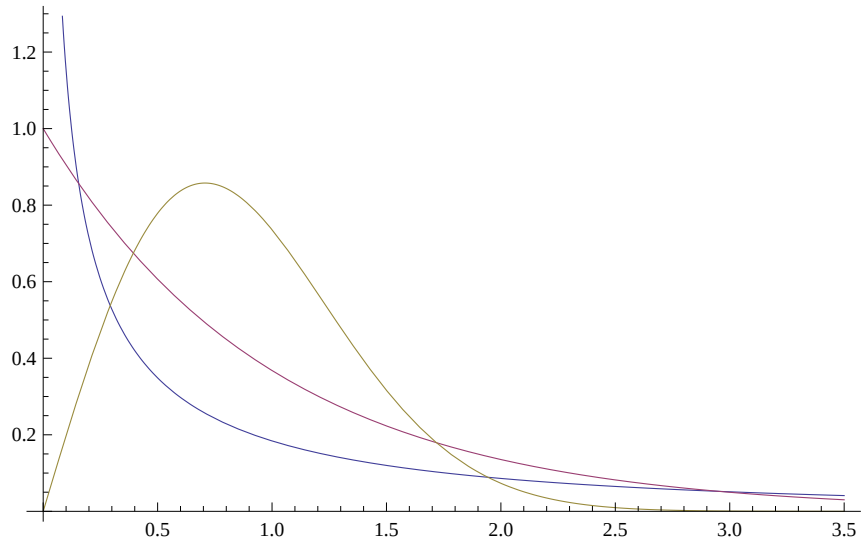
Red:(1, 0, 1) (exponential); Green:(1, 0, 2); Cyan:(1, 2, 2); Blue:(1, 2, 4)

Three Parameter Weibull: Applications

$$f(x;\alpha,\beta,\gamma) = \begin{cases} \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1} e^{-((x-\beta)/\alpha)^{\gamma}} & \text{if } x \geq \beta \\ 0 & \text{otherwise.} \end{cases}$$

Arises in many places, such as survival analysis.

- $\gamma < 1$: high infant mortality;
- $\gamma = 1$: constant failure rate;
- $\gamma > 1$: aging process.



The Gamma Distribution and Weibulls

- For $s > 0$, define the Γ-function by

$$\Gamma(s) = \int_0^\infty e^{-u} u^{s-1} \mathrm{d}u = \int_0^\infty e^{-u} u^s \frac{\mathrm{d}u}{u}.$$

- Generalizes factorial function: $\Gamma(n) = (n-1)!$ for $n \geq 1$ an integer.

A Weibull distribution with parameters α, β, γ has:

- Mean: $\alpha\Gamma\left(1 + 1/\gamma\right) + \beta$.
- Variance: $\alpha^2\Gamma\left(1 + 2/\gamma\right) - \alpha^2\Gamma\left(1 + 1/\gamma\right)^2$.

Weibull Integrations

$$\begin{aligned}\mu_{\alpha,\beta,\gamma} &= \int_\beta^\infty x \cdot \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1} e^{-((x-\beta)/\alpha)^\gamma}\,\mathrm{d}x \\ &= \int_\beta^\infty \alpha\frac{x-\beta}{\alpha} \cdot \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1} e^{-((x-\beta)/\alpha)^\gamma}\,\mathrm{d}x \; + \; \beta.\end{aligned}$$

Change variables: $u = \left(\frac{x-\beta}{\alpha}\right)^\gamma$, so $\mathrm{d}u = \frac{\gamma}{\alpha}\left(\frac{x-\beta}{\alpha}\right)^{\gamma-1}\mathrm{d}x$ and

$$\begin{aligned}\mu_{\alpha,\beta,\gamma} &= \int_0^\infty \alpha u^{1/\gamma} \cdot e^{-u}\mathrm{d}u \; + \; \beta \\ &= \alpha\int_0^\infty e^{-u}\, u^{1+1/\gamma}\,\frac{\mathrm{d}u}{u} \; + \; \beta \\ &= \alpha\Gamma(1+1/\gamma) \; + \; \beta.\end{aligned}$$

A similar calculation determines the variance.

The Pythagorean Theorem

American League

Select favorite team | Standings as of | Jun | 5 | 2008 | Go

East	W	L	PCT	GB	L10	STRK	INT	HOME	ROAD	X W-L	LAST GAME	NEXT GAME
Boston	37	25	.597	-	6-4	W2	3-0	23-5	14-20	36-26	6/4 v TB, W 5-1	6/5 v TB, 6:05P
Tampa Bay	35	24	.593	0.5	6-4	L2	1-2	24-10	11-14	32-27	6/4 @ BOS, L 1-5	6/5 @ BOS, 6:05P
Toronto	32	29	.525	4.5	6-4	L1	2-1	15-11	17-18	34-27	6/4 @ NYY, L 1-5	6/5 @ NYY, 1:05P
New York	29	30	.492	6.5	5-5	W1	0-2	15-13	14-17	28-31	6/4 v TOR, W 5-1	6/5 v TOR, 1:05P
Baltimore	28	30	.483	7.0	4-6	L1	2-1	17-11	11-19	27-31	6/4 @ MIN, L 5-7	6/5 @ MIN, 1:10P

Central	W	L	PCT	GB	L10	STRK	INT	HOME	ROAD	X W-L	LAST GAME	NEXT GAME
Chicago	32	26	.552	-	6-4	W2	3-0	15-9	17-17	34-24	6/4 v KC, W 6-4	6/5 v KC, 8:11P
Minnesota	31	28	.525	1.5	7-3	W1	1-2	19-15	12-13	29-30	6/4 v BAL, W 7-5	6/5 v BAL, 1:10P
Cleveland	27	32	.458	5.5	4-6	W1	0-3	16-16	11-16	31-28	6/4 @ TEX, W 15-9	6/5 @ TEX, 8:05P
Detroit	24	35	.407	8.5	3-7	L3	1-2	12-14	12-21	27-32	6/4 @ OAK, L 2-10	6/6 v CLE, 7:05P
Kansas City	23	36	.390	9.5	2-8	L2	2-1	12-16	11-20	23-36	6/4 @ CWS, L 4-6	6/5 @ CWS, 8:11P

West	W	L	PCT	GB	L10	STRK	INT	HOME	ROAD	X W-L	LAST GAME	NEXT GAME
Los Angeles	37	24	.607	-	7-3	W5	2-1	18-13	19-11	31-30	6/4 @ SEA, W 5-4	6/6 @ OAK, 10:05P
Oakland	33	27	.550	3.5	6-4	W4	1-2	20-13	13-14	35-25	6/4 v DET, W 10-2	6/6 v LAA, 10:05P
Texas	30	31	.492	7.0	5-5	L1	2-1	15-14	15-17	29-32	6/4 v CLE, L 9-15	6/5 v CLE, 8:05P
Seattle	21	39	.350	15.5	3-7	L4	2-1	14-19	7-20	24-36	6/4 v LAA, L 4-5	6/6 @ BOS, 7:05P

National League

East	W	L	PCT	GB	L10	STRK	INT	HOME	ROAD	X W-L	LAST GAME	NEXT GAME
Philadelphia	35	26	.574	-	8-2	L1	1-2	20-13	15-13	36-25	6/4 v CIN, L 0-2	6/5 v CIN, 1:05P
Florida	32	26	.552	1.5	4-6	W1	1-2	18-12	14-14	29-29	6/4 @ ATL, W 6-4	6/5 @ ATL, 7:00P
New York	30	28	.517	3.5	7-3	W2	2-0	17-11	13-17	30-28	6/4 @ SF, W 5-3	6/5 @ SD, 10:05P
Atlanta	31	29	.517	3.5	4-6	L1	2-1	24-8	7-21	35-25	6/4 v FLA, L 4-6	6/5 v FLA, 7:00P
Washington	24	35	.407	10.0	3-7	L3	1-2	13-16	11-19	23-36	6/4 v STL, PPD	6/5 v STL, 7:10P

Central	W	L	PCT	GB	L10	STRK	INT	HOME	ROAD	X W-L	LAST GAME	NEXT GAME

Pythagorean Won–Loss Formula: $\frac{\mathrm{RS}_{\mathrm{obs}}^{\gamma}}{\mathrm{RS}_{\mathrm{obs}}^{\gamma}+\mathrm{RA}_{\mathrm{obs}}^{\gamma}}$

Theorem: Pythagorean Won–Loss Formula (Miller '06)

Let the runs scored and allowed per game be two independent random variables drawn from Weibull distributions $(\alpha_{\mathrm{RS}}, \beta, \gamma)$ and $(\alpha_{\mathrm{RA}}, \beta, \gamma)$; α_{RS} and α_{RA} are chosen so that the Weibull means are the observed sample values RS and RA. If $\gamma > 0$ then the Won–Loss Percentage is $\frac{(\mathrm{RS}-\beta)^{\gamma}}{(\mathrm{RS}-\beta)^{\gamma}+(\mathrm{RA}-\beta)^{\gamma}}$.

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Take $\beta = -1/2$ (since runs must be integers).
$\mathrm{RS} - \beta$ estimates average runs scored, $\mathrm{RA} - \beta$ estimates average runs allowed.
Weibull with parameters (α, β, γ) has mean $\alpha\Gamma\left(1 + 1/\gamma\right) + \beta$.

Proof of the Pythagorean Won–Loss Formula

Let X and Y be independent random variables with Weibull distributions $(\alpha_{\mathrm{RS}}, \beta, \gamma)$ and $(\alpha_{\mathrm{RA}}, \beta, \gamma)$ respectively. To have means of $\mathrm{RS} - \beta$ and $\mathrm{RA} - \beta$ our calculations for the means imply

$$\alpha_{\mathrm{RS}} = \frac{\mathrm{RS} - \beta}{\Gamma(1 + 1/\gamma)}, \qquad \alpha_{\mathrm{RA}} = \frac{\mathrm{RA} - \beta}{\Gamma(1 + 1/\gamma)}.$$

We need only calculate the probability that X exceeds Y. We use the integral of a probability density is 1.

Proof of the Pythagorean Won–Loss Formula (cont)

$$\begin{aligned}
\text{Prob}(X > Y) &= \int_{x=\beta}^{\infty} \int_{y=\beta}^{x} f(x;\alpha_{\text{RS}},\beta,\gamma) f(y;\alpha_{\text{RA}},\beta,\gamma)\,dy\,dx \\
&= \int_{\beta}^{\infty} \int_{\beta}^{x} \frac{\gamma}{\alpha_{\text{RS}}} \left(\frac{x-\beta}{\alpha_{RS}}\right)^{\gamma-1} e^{-\left(\frac{x-\beta}{\alpha_{\text{RS}}}\right)^{\gamma}} \frac{\gamma}{\alpha_{\text{RA}}} \left(\frac{y-\beta}{\alpha_{\text{RA}}}\right)^{\gamma-1} e^{-\left(\frac{y-\beta}{\alpha_{\text{RA}}}\right)^{\gamma}} dy\,dx \\
&= \int_{x=0}^{\infty} \frac{\gamma}{\alpha_{\text{RS}}} \left(\frac{x}{\alpha_{RS}}\right)^{\gamma-1} e^{-\left(\frac{x}{\alpha_{\text{RS}}}\right)^{\gamma}} \left[\int_{y=0}^{x} \frac{\gamma}{\alpha_{\text{RA}}} \left(\frac{y}{\alpha_{\text{RA}}}\right)^{\gamma-1} e^{-\left(\frac{y}{\alpha_{\text{RA}}}\right)^{\gamma}} dy\right] dx \\
&= \int_{x=0}^{\infty} \frac{\gamma}{\alpha_{\text{RS}}} \left(\frac{x}{\alpha_{RS}}\right)^{\gamma-1} e^{-(x/\alpha_{\text{RS}})^{\gamma}} \left[1 - e^{-(x/\alpha_{\text{RA}})^{\gamma}}\right] dx \\
&= 1 - \int_{x=0}^{\infty} \frac{\gamma}{\alpha_{\text{RS}}} \left(\frac{x}{\alpha_{RS}}\right)^{\gamma-1} e^{-(x/\alpha)^{\gamma}} dx,
\end{aligned}$$

where we have set

$$\frac{1}{\alpha^{\gamma}} = \frac{1}{\alpha_{\text{RS}}^{\gamma}} + \frac{1}{\alpha_{\text{RA}}^{\gamma}} = \frac{\alpha_{\text{RS}}^{\gamma} + \alpha_{\text{RA}}^{\gamma}}{\alpha_{\text{RS}}^{\gamma}\alpha_{\text{RA}}^{\gamma}}.$$

Proof of the Pythagorean Won–Loss Formula (cont)

$$
\begin{aligned}
\text{Prob}(X > Y) &= 1 - \frac{\alpha^\gamma}{\alpha_{\text{RS}}^\gamma} \int_0^\infty \frac{\gamma}{\alpha} \left(\frac{x}{\alpha}\right)^{\gamma-1} e^{(x/\alpha)^\gamma} \, dx \\
&= 1 - \frac{\alpha^\gamma}{\alpha_{\text{RS}}^\gamma} \\
&= 1 - \frac{1}{\alpha_{\text{RS}}^\gamma} \frac{\alpha_{\text{RS}}^\gamma \alpha_{\text{RA}}^\gamma}{\alpha_{\text{RS}}^\gamma + \alpha_{\text{RA}}^\gamma} \\
&= \frac{\alpha_{\text{RS}}^\gamma}{\alpha_{\text{RS}}^\gamma + \alpha_{\text{RA}}^\gamma}.
\end{aligned}
$$

Proof of the Pythagorean Won–Loss Formula (cont)

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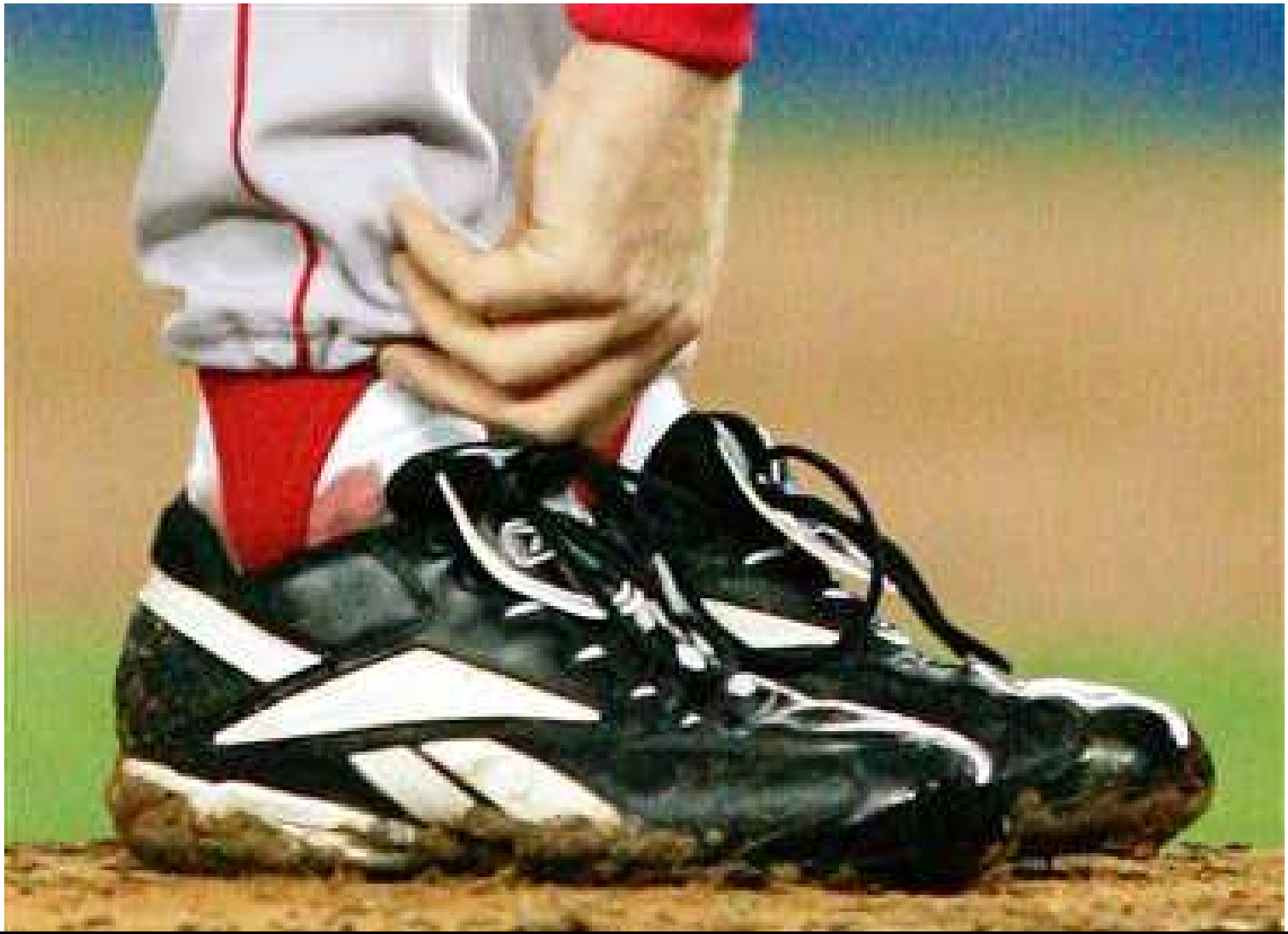
&= 1 - \frac{\alpha^\gamma}{\alpha_{\text{RS}}^\gamma} \\
&= 1 - \frac{1}{\alpha_{\text{RS}}^\gamma} \frac{\alpha_{\text{RS}}^\gamma \alpha_{\text{RA}}^\gamma}{\alpha_{\text{RS}}^\gamma + \alpha_{\text{RA}}^\gamma} \\
&= \frac{\alpha_{\text{RS}}^\gamma}{\alpha_{\text{RS}}^\gamma + \alpha_{\text{RA}}^\gamma}.
\end{aligned}$$

We substitute the relations for α_{RS} and α_{RA} and find that

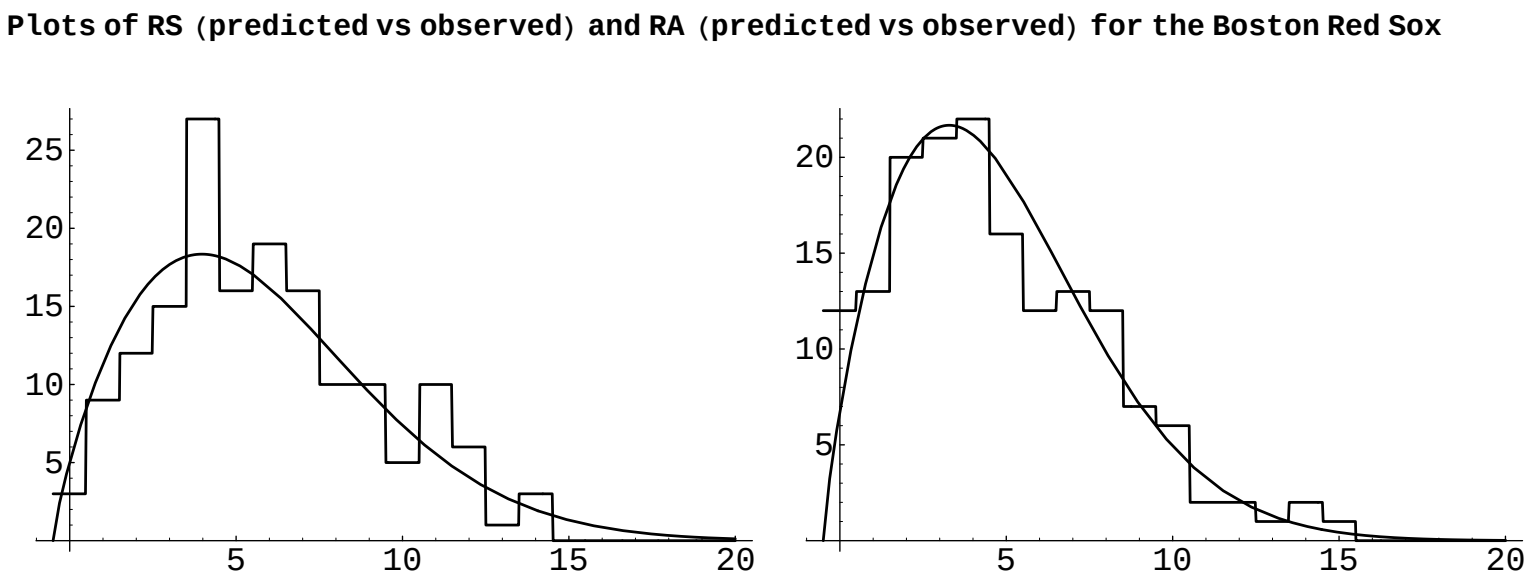
$$\text{Prob}(X > Y) = \frac{(\text{RS} - \beta)^\gamma}{(\text{RS} - \beta)^\gamma + (\text{RA} - \beta)^\gamma}.$$

Note $\text{RS} - \beta$ estimates RS_{obs}, $\text{RA} - \beta$ estimates RA_{obs}.

Analysis of 2004



Best Fit Weibulls to Data (Method of Maximum Likelihood)



Using as bins $[-.5, .5] \cup [.5, 1.5] \cup \cdots \cup [7.5, 8.5] \cup [8.5, 9.5] \cup [9.5, 11.5] \cup [11.5, \infty)$.

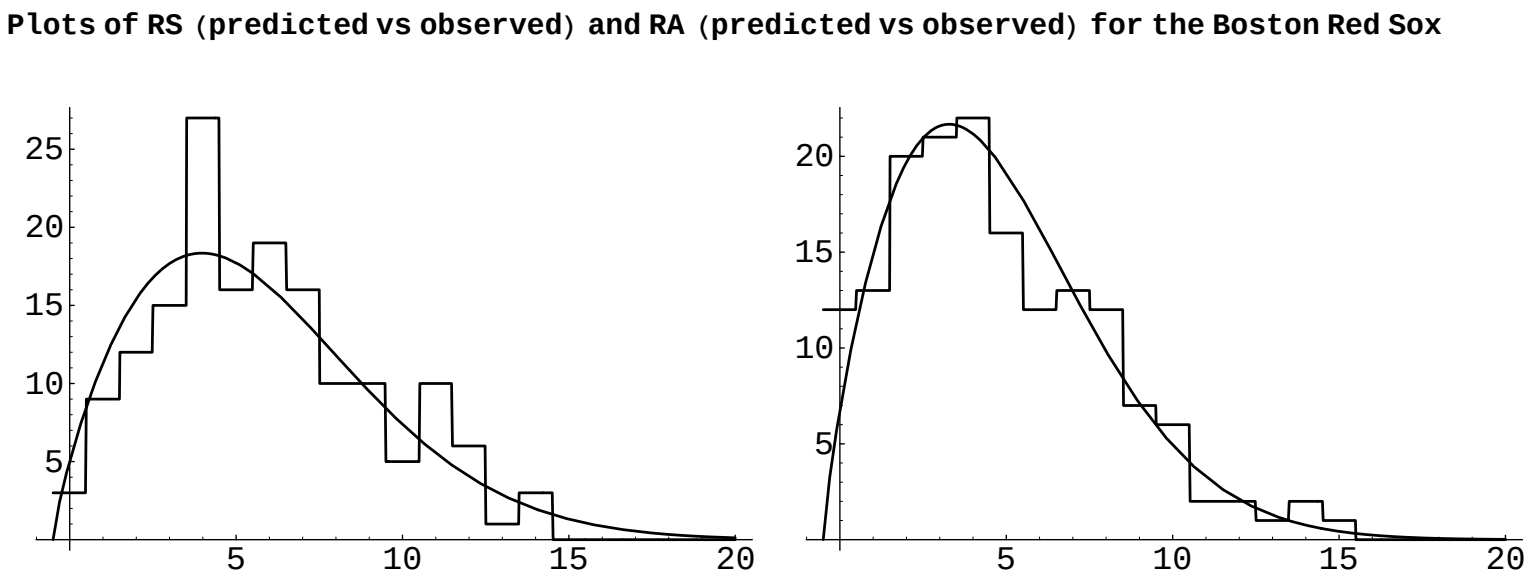
Best Fit Weibulls to Data: Method of Least Squares

- $\mathrm{Bin}(k)$ is the k^{th} bin;
- $\mathrm{RS}_{\mathrm{obs}}(k)$ (resp. $\mathrm{RA}_{\mathrm{obs}}(k)$) the observed number of games with the number of runs scored (allowed) in $\mathrm{Bin}(k)$;
- $A(\alpha, \gamma, k)$ the area under the Weibull with parameters $(\alpha, -1/2, \gamma)$ in $\mathrm{Bin}(k)$.

Find the values of $(\alpha_{\mathrm{RS}}, \alpha_{\mathrm{RA}}, \gamma)$ that minimize

$$\sum_{k=1}^{\#\mathrm{Bins}} \left(\mathrm{RS}_{\mathrm{obs}}(k) - \#\mathrm{Games} \cdot A(\alpha_{\mathrm{RS}}, \gamma, k)\right)^2 + \sum_{k=1}^{\#\mathrm{Bins}} \left(\mathrm{RA}_{\mathrm{obs}}(k) - \#\mathrm{Games} \cdot A(\alpha_{\mathrm{RA}}, \gamma, k)\right)^2 .$$

Best Fit Weibulls to Data (Method of Maximum Likelihood)



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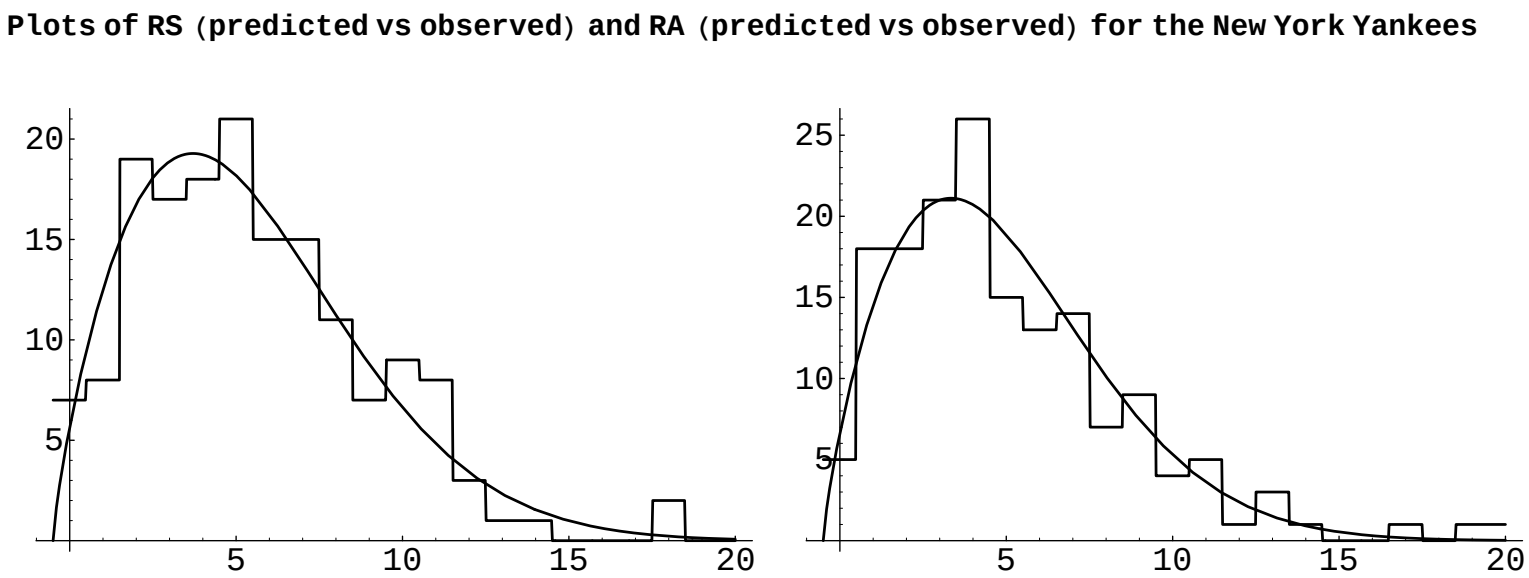

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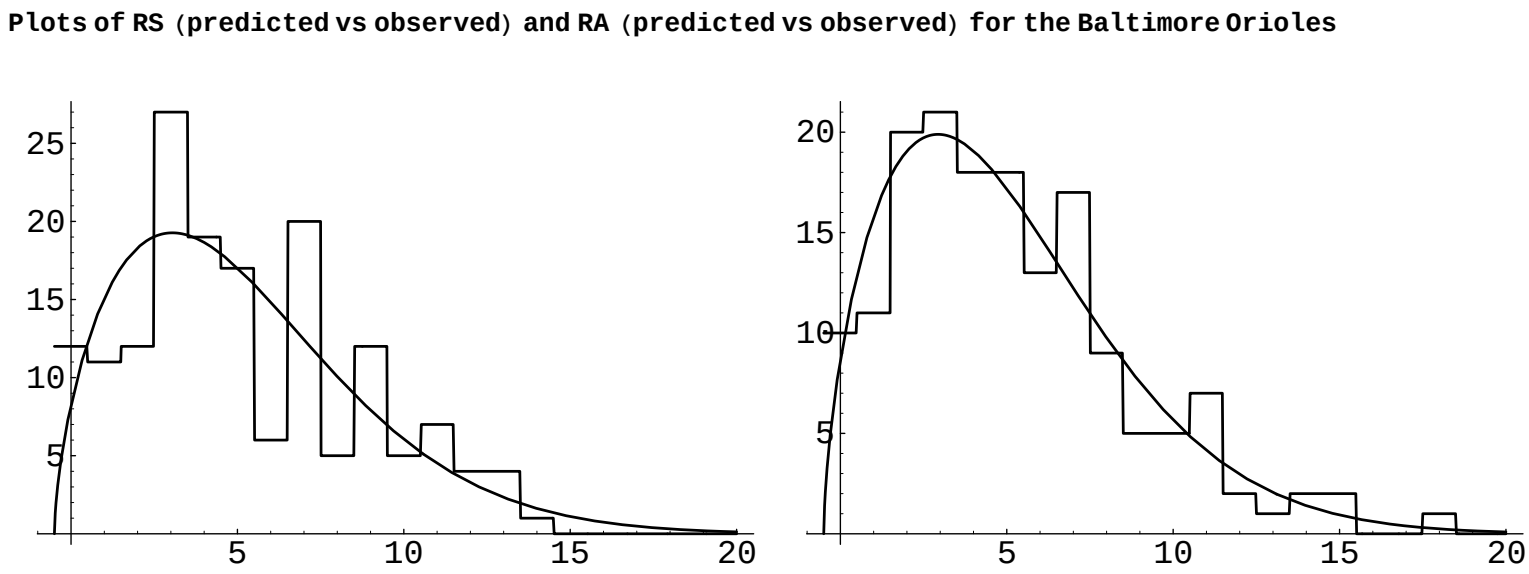

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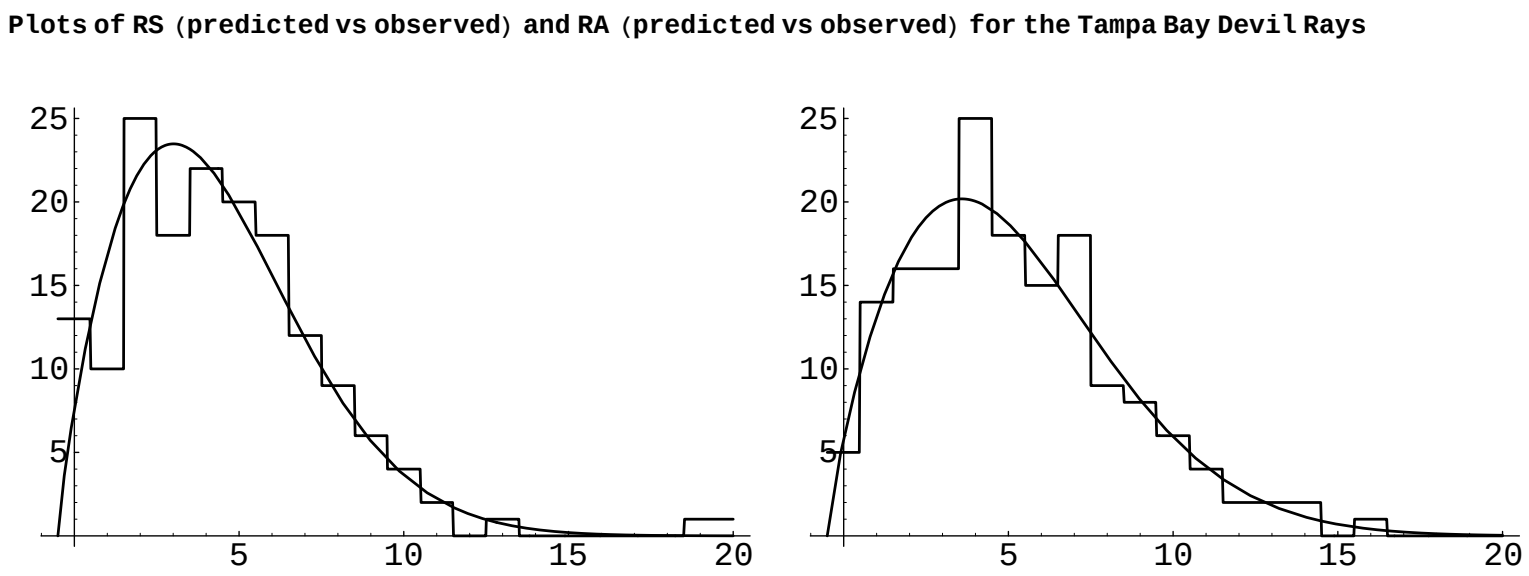

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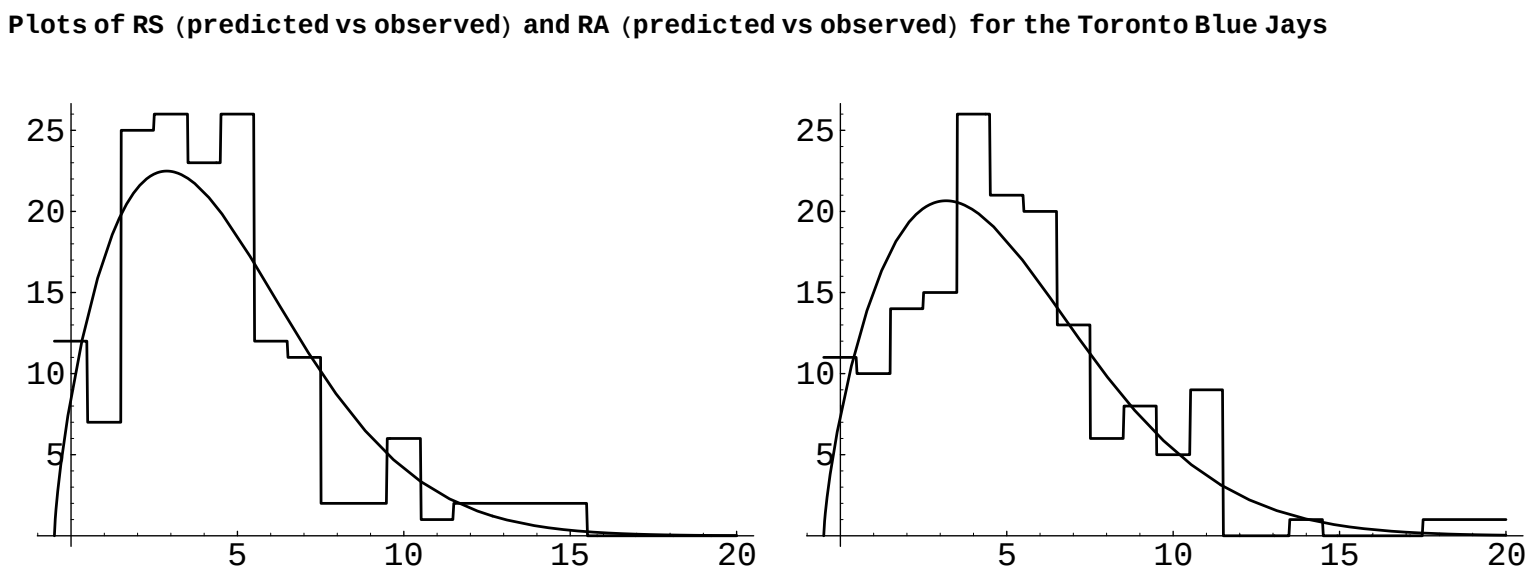

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Advanced Theory

Bonferroni Adjustments

Fair coin: 1,000,000 flips, expect 500,000 heads.

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Consider *N* independent experiments of flipping a fair coin 1,000,000 times. *What is the probability that at least one of set doesn't have* $499,000 \leq \#\text{Heads} \leq 501,000$*?*

N	Probability
5	22.62
14	51.23
50	92.31

See unlikely events happen as *N* increases!

Data Analysis: χ^2 Tests (20 and 109 degrees of freedom)

Team	RS+RA χ2: 20 d.f.	Indep χ2: 109 d.f
Boston Red Sox	15.63	83.19
New York Yankees	12.60	129.13
Baltimore Orioles	29.11	116.88
Tampa Bay Devil Rays	13.67	111.08
Toronto Blue Jays	41.18	100.11
Minnesota Twins	17.46	97.93
Chicago White Sox	22.51	153.07
Cleveland Indians	17.88	107.14
Detroit Tigers	12.50	131.27
Kansas City Royals	28.18	111.45
Los Angeles Angels	23.19	125.13
Oakland Athletics	30.22	133.72
Texas Rangers	16.57	111.96
Seattle Mariners	21.57	141.00

20 d.f.: 31.41 (at the 95% level) and 37.57 (at the 99% level).
109 d.f.: 134.4 (at the 95% level) and 146.3 (at the 99% level).
Bonferroni Adjustment:
20 d.f.: 41.14 (at the 95% level) and 46.38 (at the 99% level).
109 d.f.: 152.9 (at the 95% level) and 162.2 (at the 99% level).

Data Analysis: Structural Zeros

- For independence of runs scored and allowed, use bins $[0,1) \cup [1,2) \cup [2,3) \cup \cdots \cup [8,9) \cup [9,10) \cup [10,11) \cup [11,\infty)$.
- Have an $r \times c$ contingency table with structural zeros (runs scored and allowed per game are never equal).
- (Essentially) $O_{r,r} = 0$ for all r, use an iterative fitting procedure to obtain maximum likelihood estimators for $E_{r,c}$ (expected frequency of cell (r, c) assuming that, given runs scored and allowed are distinct, the runs scored and allowed are independent).

Summary

Testing the Model: Data from Method of Maximum Likelihood

Team	Obs Wins	Pred Wins	ObsPerc	PredPerc	GamesDiff	γ
Boston Red Sox	98	93.0	0.605	0.574	5.03	1.82
New York Yankees	101	87.5	0.623	0.540	13.49	1.78
Baltimore Orioles	78	83.1	0.481	0.513	-5.08	1.66
Tampa Bay Devil Rays	70	69.6	0.435	0.432	0.38	1.83
Toronto Blue Jays	67	74.6	0.416	0.464	-7.65	1.97
Minnesota Twins	92	84.7	0.568	0.523	7.31	1.79
Chicago White Sox	83	85.3	0.512	0.527	-2.33	1.73
Cleveland Indians	80	80.0	0.494	0.494	0.	1.79
Detroit Tigers	72	80.0	0.444	0.494	-8.02	1.78
Kansas City Royals	58	68.7	0.358	0.424	-10.65	1.76
Los Angeles Angels	92	87.5	0.568	0.540	4.53	1.71
Oakland Athletics	91	84.0	0.562	0.519	6.99	1.76
Texas Rangers	89	87.3	0.549	0.539	1.71	1.90
Seattle Mariners	63	70.7	0.389	0.436	-7.66	1.78

γ: mean $= 1.74$, standard deviation $= .06$, median $= 1.76$;
close to numerically observed value of 1.82.

Conclusions

- Find parameters such that Weibulls are good fits;
- Runs scored and allowed per game are statistically independent;
- Pythagorean Won–Loss Formula is a consequence of our model;
- Best γ (both close to observed best 1.82):
  - ◇ Method of Least Squares: 1.79;
  - ◇ Method of Maximum Likelihood: 1.74.

Future Work

- Micro-analysis: runs scored and allowed aren't independent (big lead, close game), run production smaller for inter-league games in NL parks, ...
- Other sports: Does the same model work? Basketball has γ between 14 and 16.5.
- Closed forms: Are there other probability distributions that give integrals which can be determined in closed form?
- Valuing Runs: Pythagorean formula used to value players (10 runs equals 1 win); better model leads to better team.

Smoots

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Oliver Smoot: Chairman of the American National Standards Institute (ANSI) from 2001 to 2002, President of the International Organization for Standardization (ISO) from 2003 to 2004.

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  ◇ *Relieving and Readjusting Pythagoras* (senior thesis of Victor Luo, 2014).
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