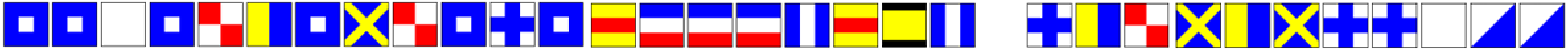

Math/Stat Thesis Showcase

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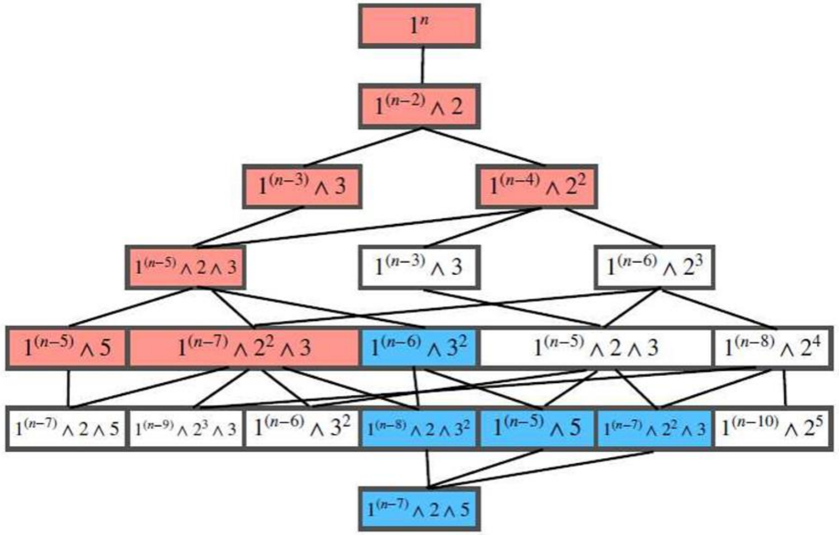
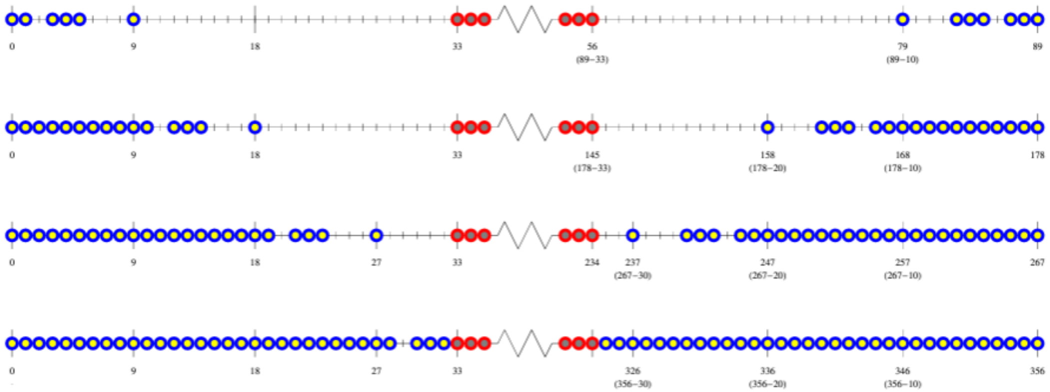
http://www.williams.edu/Mathematics/sjmiller/public_html





The Riemann Zeta Function ζ(s)

For s > 1:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \ldots$$

(more generally: s any complex number with real part at least 1).

Looking at this function, NOT clear why it is worth studying....

Integers and Primes

Most of us are familiar with the positive integers: 1, 2, 3, 4, 5, ….

What is the next integer after 2024?

Integers and Primes

Most of us are familiar with the positive integers: 1, 2, 3, 4, 5, ….

What is the next integer after 2024? 2025

Not much mystery in the spacings between integers!

Explicit formula for n^{th} integer….

Integers and Primes

What about the primes: 2, 3, 5, 7, ….

What is the next prime after 2024?

Integers and Primes

What about the primes: 2, 3, 5, 7, ….

What is the next prime after 2024? 2027

What is the next prime after 2027?

Integers and Primes

What about the primes: 2, 3, 5, 7, ….

What is the next prime after 2024? 2027

What is the next prime after 2027? 2029

Lot harder to find the next prime than to find the next integer!

The Riemann Zeta Function ζ(s) and Primes

We defined the Riemann Zeta Function (for s > 1) by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \ldots$$

and now we note a remarkable property; we also have

$$\zeta(s) = \prod_{p \text{ prime}} \left(1 - \frac{1}{p^s}\right)^{-1} = \left(1 - \frac{1}{2^s}\right)^{-1} \left(1 - \frac{1}{3^s}\right)^{-1} \left(1 - \frac{1}{5^s}\right)^{-1} \ldots .$$

Two questions: (1) Why is this true, and (2) Why do we care?

The Riemann Zeta Function ζ(s) and Primes

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \left(1 - \frac{1}{p^s}\right)^{-1},$$

- If we take s=1 the sum becomes the Harmonic Series, diverges!

- If there were only finitely many primes the product is rational, but $\zeta(2) = \frac{\pi^2}{6}$

Proper Subset of Other Topics:

- Sabermetics
- Additive Number Theory
- Probability (Benford's Law of Digit Bias, German Tank Problem)
- Erdos Distance Problem
- Operations Research….