

Teaching Experience and Teaching Statement

Steven J. Miller (Williams College, sjm1@williams.edu)

Contents

1	Summary	1
2	Providing Opportunities	5
3	Standard Classes	6
3.1	Williams College	6
3.2	Brown University	7
3.3	The Ohio State University	8
3.4	Princeton University	9
4	Independent Seminars	9
5	Undergraduate and Graduate Research	10
6	Mathematics Education	11
7	VIGRE Undergraduate Mathematics Laboratory	11
7.1	Purpose	11
7.2	Course Structure	12
7.3	Topics and Results	12
7.4	My Duties	13
7.5	Summary	13

1 Summary

Of all the classes I've taught, the two most reflective of my classroom teaching philosophy are the Undergraduate Mathematics Laboratory (to be described immediately below) at Princeton and Probability/Statistics (to be described later) at Brown and now Williams. My main goals as an educator are to teach students the *techniques* that they can use over and over (both for theoretical studies as well as practical applications), as well as to provide *opportunities* (which ranges from finding novel paths to excite them about the material to giving them the chance to work on problems for major firms). There are always trade-offs; while I cover slightly less material than I might wish, I am able to highlight the connections to mathematics and show them how these may be used for a variety of real world problems. I frequently chat with former students to find out what material and mindsets they use, as well as what they wish they had learned, and I incorporate these in my classes. I also have extensive contacts in industry (which I use to bring guest speakers to class), and I have worked with students on projects for the IRS and a major league baseball team, as well as bringing the students on as contributors to books I am writing. This allows me to give my students the rare opportunity of seeing firsthand certain careers.

I designed the Undergraduate Mathematics Laboratory with Peter Sarnak as part of Princeton's VIGRE grant to foster more collaboration between undergraduates, graduate students, postdocs and faculty. I ran the course four times there, once each with Peter Sarnak, Andrew Wiles, Ramin Takloo-Bighash, and Yakov Sinai. This course allowed juniors to investigate current unsolved conjectures theoretically and numerically. I have taught variants of this course at the Courant Institute, NYU, with Peter Sarnak and Alex Barnett, a similar course (primarily for graduate students) with Vitaly Bergelson and Warren Sinnott at The Ohio State University, and four advanced undergraduate research class at Brown (one with Jill Pipher). I have run a summer version twice, by myself in 2004 at Ohio State and in 2003 at AIM with Brian Conrey, David Farmer, Chris Hughes and Mike Rubinstein. I incorporate features of these courses in the more standard courses I teach. These classes often have the last few weeks devoted to advanced topics chosen in conjunction with the students, with research opportunities presented and encouraged throughout the semester. I have found students enjoy the empowerment this provides them, and respond beautifully to such challenges.

I love both mathematical research and teaching, and the research classes and programs are an ideal way to combine the two. While we use computers in the class, they are used as a tool to interest students in mathematics; very quickly, students are able to see phenomena no one has seen before, and we have found this sense of discovery and ownership greatly increases students' interest and motivation. While such classes take a lot of time, they offer a real chance at making a difference in someone's mathematical career, and allow me to explore a variety of topics. Students see the problems and techniques used in current research early in their studies, and are better prepared to decide their career path. While these programs have led to several joint papers with undergraduates (from graph theory to random matrix theory to elliptic curves), it is important to note that this is not the goal. The purpose is to choose interesting problems which are accessible to beginning students; over the years I have successfully supervised undergraduates ranging from entering freshmen knowing only BC calculus to advanced undergraduates preparing for graduate study.

I wrote a textbook based on these courses, *An Invitation to Modern Number Theory* (with Ramin Takloo-Bighash). The book is similar to the courses: the goal is to introduce enough background material for students to see the mathematical landscape. Topics include elementary number theory, Diophantine equations, continued fractions, probabilistic number theory, equidistribution, and L -functions and random matrix theory. Because of my extensive experience supervising undergraduate and graduate students, as well as serving on three dissertation committees (one in analysis and two in number theory), I feel ready to advise graduate students on thesis problems.

I have also taught numerous standard courses. In these classes I always provide advanced material and open research problems. Students respond to a challenge, and enjoy the opportunity to further explore the subject. Mathematics has been successfully applied to many problems in the real world, and these connections can be used to keep the students interested and motivated; this is especially important as I often have a diverse student body with enormous variations in motivation, ability and needs (ranging from some who are preparing for graduate school to others who have to satisfy the requirements of the major). Examples from my classes include cryptography (Abstract Algebra), scheduling problems for airlines (Linear Algebra / Linear Programming), and determining why certain baseball statistics predict a team's performance (Mathematical Statistics). While detailed descriptions of all the courses are provided later, one course I taught at Brown and one at Williams capture my teaching philosophy and style, and are briefly summarized below.

I taught Math 162 (Mathematical Statistics) for four years at Brown. This is a theory intensive statistics course. In addition to proving standard results such as the central limit theorem (assuming a standard result from complex analysis), we introduce numerous useful proof techniques and explore their applications. For example, we discuss differentiating identities and matching coefficients and show applications to formulas for tests based on runs. The course is peppered with numerous applied problems with interesting theoretical components (many of the examples come from student projects, a required and popular component of the class). Examples range from analysis of baseball games (where the fact that games cannot end in ties led us to the concept of structural zeros in $r \times c$ tables) to Benford's law and digit bias (which is related to important problems in equidistribution theory, and is currently being used by the IRS to detect corporate tax fraud). In my four years at Brown, this was the top rated mathematics course twice, and the second highest ranked course another year.

In Fall 2009 and 2012 I taught Math 341 (Probability) at Williams. In many respects this course is similar to the statistics course I taught at Brown; the differences, however, highlight my growth as an educator and are the result of what I have learned from years of teaching. We frequently encounter students with varied levels of background, interest, and time that they choose to devote to our courses. As such, we have to adjust our teaching to reach these different levels. I have added several innovations since arriving at Williams to further these goals; see the course homepage http://www.williams.edu/Mathematics/sjmillier/public_html/341/index.htm for more details.

- After each class I spend 30 to 90 minutes typing up additional comments on the lecture. These range from summarizing key points to providing advanced theory and links for students who want to explore further. I am heartened by how many students at all levels and interests tell me they read these posts. I have also written many extensive handouts giving (almost) complete details of the advanced theory (for example, moment generating functions and the needed results from complex analysis).
- I post detailed beamer notes of the lectures online; during lecture I use these notes as a starting point for discussions of the theory, and then move to the blackboard for the detailed computations.
- I have started using clickers in my classes to get immediate feedback on how well the students understand the material. I design multiple choice questions whose answers advance the theory and class discussion, which we then vote on. Recent examples include questions on the rate of convergence to the standard normal to mathematical gambling to detecting fraud.
- I encourage all my students to do a project, which can count for 10% of their grade; many take me up on the offer and explore a subject in great detail.
- Occasionally my students are asked to create two homework problems on an assignment, one where they must include a solution and one where they pose a problem for me to solve. I have found these a wonderful way to tap into the abilities, interests and creativity of my students. They learn how difficult (but important) it is to formulate a problem clearly and unambiguously; further, this gives me opportunity for personalized feedback to them on how well they know the material. Other innovations in the homework assignments include the option of writing a scientific summary of a paper or giving a small presentation to me and part of the class instead of additional standard problems. At the end of the day the goal is not to train students to solve homework problems, but to apply what they've learned to the real world and to do original research; these projects help give students a flavor of what this is like.

- In the 2012 iteration of the class, the textbook is ‘The Probability Lifesaver’, a book I am writing for Princeton University Press as a sequel of sorts to their successful ‘The Calculus Lifesaver’. Several students have had significant roles in writing this book, ranging from reviewing the exposition to writing sections of the book. This has been a tremendous experience for the students, giving them a valuable perspective, and has helped keep the book clear and accessible.

Over the past 10 years, I have supervised almost 200 undergraduate and graduate students on various projects. While many of these have led to publications, that is not the point (for undergraduate research!). There, for me the purpose is to show students what research is like, and to give them a sense of what problems we study, what techniques we use, and whether or not this is something they wish to pursue. This is not limited to my thesis and summer students, and is also a part of my classes (for example, in my last four courses above multivariable calculus, each will lead to a publication with a student on a research problem, or a solution to a problem for someone in industry).

I have served on three graduate dissertation committees, and been a major advisor to two other theses. I have continued working and publishing with several of my undergraduate students after they’ve begun their graduate studies. Based on my prior experience with undergraduate and graduate students, I am confident about supervising a PhD dissertation.

I have been the faculty advisor to the undergraduate math clubs at Ohio State and Brown University, and the Math Team at Williams. Duties range from giving lectures to helping them organize student conferences to running Putnam sessions to teaching them how to use LaTeX and programming environments such as Mathematica and Matlab to advising them on how to write papers and give talks (for example, in the past three years at least 30 of my undergraduates will give talks at research conferences).

Finally, I am very active in Mathematics Education. I write and review problems for the AMC competitions. I am a frequent participant at meetings at the Institute for Mathematics & Education at the University of Arizona. I have lectured at many of the top programs for talented high school students (the Ross program at Ohio State, the PROMYS program at BU, and Hampshire College); for two of the past four summers I also served as a research mentor at PROMYS. I also maintain a math riddles webpage,

<http://mathriddles.williams.edu/>

(which is currently one of the top hits when googling ‘math riddles’). The purpose of this site is to help students and teachers explore mathematics through fun riddles. It receives over 4000 distinct hits a month and is used in classrooms around the world; I am working on expanding the features with students and teachers. Additionally, for the past three years I have taught continuing education classes to high school and middle school teachers at the Teachers As Scholars program in Boston. Expanding on this, I am participating in the Value of Computational Thinking across Grade Levels (VCTAL) Project, writing a unit for high school / junior high school classes on cryptography and another on streaming video. Finally, I have two educational books under contract, one on cryptography and one on probability; both of these have student involvement (a third, on linear programming, will be done after these projects are finished).

2 Providing Opportunities

I see a major part of my job as providing opportunities. This ranges from tailoring the presentation of material to each students' interests to showing them what one can do with a mathematics background. Everyone has different passions, and I try hard to find a way to incorporate these in the class. Students respond beautifully, often doing an incredible amount of work and exploring material in great detail.

In the past few years, here are some of the opportunities I have been able to provide to my students:

1. Collaborating with me on three books (Cryptography, Probability and Benford's Law).
2. Helping to run my Math Riddles page: <http://mathriddles.williams.edu>. In addition to the html coding and layout decisions, students are also providing input on the difficulty of the riddles and detailing their thought processes as they attack the riddles (these will be posted soon). The goal is to create a resource to help motivated students and teachers explore mathematics.
3. Industry: My students have done projects for a major league baseball franchise, the IRS, financial firms, and engineering companies, and I have had representatives of various companies come to speak to my students.
4. My senior seminar class (Fall 2010) and number theory class (Fall 2011) refereed papers for research journals, and wrote reviews for MathSciNet. Both of these items provide invaluable perspective.
5. Co-organizing conferences: Several of my students co-organized a special session on undergraduate research at an AMS sectional (at Holy Cross in 2011 and at Boston College in 2013).
6. I organized a panel on undergraduate research at the Joint Meetings in Boston in 2012, and had one of my thesis students serve as a panelist. The conference report is online at

http://web.williams.edu/Mathematics/sjmilller/public_html/math/handouts/NotesonPanelonUndergraduateResearch12.pdf

In each class I teach, I talk with my students to discover why they are taking the class, what they want to get out of both the class and their education, and what their future plans are. I use this to chose special topics for the end of the semester, as well as for the additional comments I write after each class.

These comments are designed to help students explore the material further, emphasizing either interesting points from the lecture or connections with mathematics and other fields. While these comments take awhile to write, from anonymous surveys I have been told that a large amount of the class reads a good number of these. Many of my students will swing by my office to discuss items raised here in greater depth. As there is only a limited amount of time available in class, these provide a good way for me to continue the conversation. Representative samples can be seen online at

1. Complex Analysis:
http://www.williams.edu/Mathematics/sjmilller/public_html/302/addcomments.htm
2. Probability:
http://www.williams.edu/Mathematics/sjmilller/public_html/341/addcomments.htm
3. Analysis and Number Theory:
http://www.williams.edu/Mathematics/sjmilller/public_html/406/discussions/index.htm

4. Differential Equations:
http://www.williams.edu/Mathematics/sjmiller/public_html/209/discussions/index.htm
5. Multivariable Calculus:
http://www.williams.edu/Mathematics/sjmiller/public_html/105/addcomments.htm

3 Standard Classes

I've taught the following subjects (detailed descriptions below):

1. Complex Analysis.
2. Number Theory: advanced senior seminar and introductory versions.
3. Linear Programming: advanced senior seminar and introductory versions.
4. Linear algebra: graduate linear algebra, honors linear algebra, linear algebra.
5. Probability.
6. Statistics: mathematical statistics.
7. Abstract Algebra.
8. Discrete mathematics.
9. Calculus III: honors multivariable calculus, multivariable calculus.
10. Calculus I.
11. Seminars: freshmen seminar (from riddles to modern mathematics), reading classics working group (on Euler), fractal geometry working group.
12. Problem solving: honors problem solving, advanced honors problem solving.
13. Pre-calculus: fundamentals of college algebra.
14. Basic College Mathematics.

3.1 Williams College

2013, 2011, 2010: Math 105: Multivariable Calculus. Special topics include statistics (method of least squares), Monte Carlo integration, sabermetrics.

2013, 2010: Math 10: Lqwurgxfwlrq wr Fubswrjudskb (Introduction to Cryptography). This is a three week winter study exploration in cryptography. Students come from a variety of backgrounds, from English majors interested in the history to math majors excited about delving into the theory.

2012, 2009: Math 341: Probability. In addition to the standard first semester topics, we discussed complex analysis in detail (so that students would have a sense of what is needed to prove the central limit theorem in general), some advanced real analysis, and current research papers in additive number theory, dynamics, auditing and random matrix theory, to name a few. Main difference in 2012 is student involvement in the book I'm writing.

2012: Math 416: Advanced Applied Linear Algebra: Main topics are Linear Programming and Random Matrix Theory. Students had to do a Linear Programming research project / presentation, where they were required to read a paper in the field dealing with a technical issue or problem and present to the class, or use the standard methods to solve a real world problem (in some cases for actual customers).

2011 – 2012: I was on sabbatical. I taught number theory at Smith, and probability and then an independent study on linear programming at Mount Holyoke.

2010: Math 308 / 406: Analysis and Number Theory. This is a tutorial; students work in groups of two and meet with me once a week to discuss the material. Students have great freedom in choice of topics. Some have chosen to pursue research projects (similar to the Undergraduate Math Laboratory at Princeton), and presented at a research conference. All students wrote reviews for MathSciNet, and refereed original research papers for journals. Both of these help the students gain a better perspective of mathematics.

2010: Math 302: Complex Analysis: This course builds on our real analysis class. We moved at a rapid pace, covering topics ranging from the residue calculus to a proof of the Riemann mapping theorem. Many students are interested in pursuing the material in greater depth; several of us meet once a week for advanced lectures, and a few students are working on research projects (one, a real analogue of the Schwarz lemma, will be published in the Monthly).

2010: Problem Solving: This is an independent study on problem solving, designed to prepare our students for both the Putnam competition and critical thinking.

2009: Math 406: Number Theory. This was a graduate level class, very similar to the lecture components of the Undergraduate Mathematics Laboratory but without the research projects.

2009: Math 209: Differential Equations. In this course we introduce some of the proof techniques of higher mathematics. I chose to start with difference equations and constantly referred back to them throughout the semester. We had numerous applications, my favorite being an analysis of Nelson's strategy in the Battle of Trafalgar. Students voted on the topics for the last two weeks (winners were Chaos Theory, Laplace Transforms, Calculus of Variations, and Mathematical Modeling).

2008: Math 103: Calculus I.

3.2 Brown University

2004: Math 9 (Introductory Calculus - one of five sections, fall semester).

2004: Math 52 (Linear Algebra – course coordinator, fall semester).

2005, 2006: Math 900-3B: How to think the Calculus way (summer). This is a first semester calculus course for high school students. Teaching beginner's calculus (especially to students with poor mathematics backgrounds) was very different from the other classes I've done. Each time I had an extremely bimodal class. Half the students could not do basic algebra, while the other half had already seen much of calculus. It was a real challenge designing the lectures and problem sets for such an audience.

2005, 2006: Math 153 (Abstract Algebra, fall semester). This is the only advanced course required

for undergraduate math majors at Brown. The class is typically bimodal, and the material and course must be structured to fit the needs of each group. Applications include number theory and cryptography.

2005, 2006, 2007, 2008: Math 162 (Mathematical Statistics, spring semester). This was one of my favorite classes to teach, as so many different topics can be connected to probability and statistics. We covered the basic theory of distributions and numerous statistical tests. This class was a mix of theory (we developed enough math to give an almost complete proof of the Central Limit Theorem, as well as an analysis of runs and a myriad of other topics) and applications (each student had to do a project gathering real world data on anything of interest to themselves; topics ranged from Random Matrix Theory to how hilly Providence is).

2006: Math 1 (Freshmen Seminar: From riddles to modern mathematics). This seminar is a chance to show students what types of problems mathematicians study, and how math can be applied. Topics included origami, Rubik's cubes, soap bubbles, planetary motion, cryptography, Babylonian mathematics, combinatorics, Benford's Law and detecting tax fraud, and voting theory.

2007: Math 54 (Honors Linear Algebra, spring semester). This is another theory-intensive course; however, in addition to covering the theory we also discussed the computational aspects. Linear programming was originally a theoretical oddity; several early papers said the results are interesting, but impractical as one will never be able to compute the desired quantities in a reasonable amount of time. We used discussions of the computational problems (calculating eigenvalues, propagation of errors in solving equations, efficiently multiplying matrices) as springboards to the theory. Advanced topics included linear programming, fast multiplication, and random matrix theory.

2007: Math 35 (Honors Calculus, fall semester).

3.3 The Ohio State University

2003: Discrete Mathematical Structures (Math 566), summer quarter.

2003: Basic College Mathematics (Math 104), autumn quarter.

2003: Honors Problem Solving and Advanced Honors Problem Solving (Math 187, 487), autumn quarter: introduction to modern mathematics through problem solving, for freshman and sophomores, and juniors and seniors. In addition to the class, I ran weekly Putnam problem sessions.

2003: Working Group on Fractal Geometry (with Gerald Edgar and Larry Lindsay), autumn.

2003: Working Group on Euler (with Vitaly Bergelson and Warren Sinnott), autumn and winter quarters.

2004: Graduate Linear Algebra (Math 683L), summer quarter. Topics included linear programming and Random Matrix Theory, and introduced first year students to current research topics.

2004: The Circle Method and Diversions through Number Theory, one week course for the Ross Program (talented high school and college students), summer.

3.4 Princeton University

1996 – 1999: I was the administrator / coordinator for the Math Department’s review sessions for all classes from Introductory Calculus (Math 101) to Honors Linear Algebra (Math 204).

1997 – 1998: I was the Head Teaching Assistant / Class Coordinator for Honors Multivariable Calculus (Math 203) and Honors Linear Algebra (Math 204), and taught a section of Math 204.

1999: Upon requests from many of my former students, I was asked to TA for Discrete Mathematics (Computer Science 341).

1999, 2001, 2002: I taught Introduction to Calculus (Math 101) at Princeton’s Freshman Scholars Institute. FSI is a summer program for incoming freshmen who have weak mathematical backgrounds, yet want to pursue majors in the sciences or engineering. Classes are five to seven students, meeting every day (for six weeks) for an hour and a half.

4 Independent Seminars

Since my second semester at Brown, I have almost always run an independent seminar on topics of either my or my students’ interest. Topics include

- elliptic curves,
- cryptography,
- Benford’s law,
- differential equations and mathematical modeling,
- advanced number theory,
- graph theory,
- Benford’s law of digit bias,
- sabermetrics,
- problem solving,
- probability.

The sabermetrics class is indicative of what these courses are like. I’ve taught this multiple times in addition to my standard teaching load; in Fall of 2009 at Williams I had five very highly motivated students (who actually started working in the Spring of 2009!). The course was split between my giving standard lectures on advanced material (linear programming, difference equations, markov chains, et cetera) to discussion sessions about model formulation and progress reports by the students. They did a phenomenal job, putting in enormous amounts of effort and learning a great deal (about theory and applications) in the process. Numerous times our empirical studies served as a springboard to introduce advanced theory. By semester’s end the students wrote up their projects. Some of their conclusions run counter to current baseball intuition, but the numerics support their analysis, and these were read with interest by a major league franchise.

When possible, I try to develop contacts in industry and foster connections between them and my students. In a previous independent study on sabermetrics, some of my students received a private

tour of a major league stadium; in another (on Benford's law of digit bias, which has applications to detecting tax fraud), they met with criminal investigators of the IRS.

In the Spring of 2010 I taught an independent study on probability. There were two purposes. The first is obviously to teach students probability; the second is to work on a book on the subject which is under contract with Princeton University Press (*The Probability Lifesaver*, a follow-up to *The Calculus Lifesaver* by Adrian Banner). In addition to discussing the material with the class, the students will participate in writing chapters and designing problems. This will be a tremendous opportunity for them; further, it will help ensure the book is kept at a friendly, readable level.

One of my favorite independent studies was a Putnam preparation course I did in 2010. It was a lot of fun, and led to wonderful interactions between some of our strongest problem solvers and some of our freshmen.

5 Undergraduate and Graduate Research

In addition to the research classes I have run, I typically have between 3 and 9 undergraduates working with me over the summer (9 each year for the past two years), and several writing theses with me during the year. I have supervised almost 200 undergraduate and graduate students over the past 10 years. In the last two years, I have had four thesis students at Williams and 30 students working with me over the summer in the four years I've been at Williams. My students have presented their work at numerous conferences. These range from undergraduate oriented conferences such as the Young Mathematicians Conference at Ohio State (in the past four years my students have took first place twice, second place once, third place once, and honorable mention twice), to research conferences such as AMS special sessions, CANT (Combinatorial and Additive Number Theory) 2011, and the Maine/Québec Number Theory Conference (where for the past two years my students were the only undergraduate presenters). Three of my students spoke at an international random matrix theory conference in Japan in July 2012 (rescheduled from 2011 due to the tsunami).

I see the purpose of undergraduate research as an opportunity to let students explore mathematics, and get a better sense of what we do and whether or not this is what they would like to do. Sometimes students ask me for a specific project; more often I tell them general areas I'm interested in, and we explore together until a project crystalizes.

It is challenging to provide a good research experience to summer students. One wants to have a problem worth doing, but at the same time make sure that it has a good chance of being done in just 9 weeks. I aim for a mix; I have small projects that almost surely can be done and lead to light publications, but possess numerous interesting offshoots which we can then explore. In some summers at Williams, I start with a big group project and then fragmented the students each year into teams of one or two. This allows them to experience individual as well as group work. We've recently explored topics in random matrix theory, L -functions, graph theory, Benford's law, number theory (analytic and algebraic) and probability, with over 20 accepted papers from the past four years.

I have also mentored several graduate students over the years. This ranges supervising them in integrated research classes (with senior faculty and undergraduates) to collaborating with them on smaller projects related to their main research interests to helping supervising Duc Khiem Huynh's dissertation (University of Bristol). Duc Khiem's thesis continues a paper of mine on the behavior of zeros near the central point in families of elliptic curve L -functions. With his two thesis advisors and another colleague, we have written 3 papers; further, while a graduate student Duc Khiem stayed with me for a month here at Williams, with us splitting our time between lectures and research. Currently I am working with another graduate student in marketing on a project which is now his

thesis. I am thus confident of my ability to supervise PHD dissertations (and while at Brown served on three committees, two in number theory and one in analysis).

6 Mathematics Education

I am very active in Mathematics Education. I have given lectures at the Ross Program at Ohio State, PROMYS at BU and Hampshire College; additionally, I have been a research mentor for the past 2 years at PROMYS. I didn't know about programs like these when I was younger, but I wish I had! I frequently bring my current students with me when I lecture; it's a great experience for both them and the high school students.

I have been active in math competitions as well. I have written three out of the last five exams for the annual Williams - Middlebury Math Competition, and help write and review problems for the AMC.

I have participated in several conferences at the Institute for Mathematics & Education, and this will be my second year in a row teaching two courses (one on cryptography and one on Benford's law of digit bias) to high school and middle school teachers in MA. Along these lines, I'm participating in the Value of Computational Thinking across Grade Levels (VCTAL) Project, where I will be writing modules for schools.

I am currently revamping my math riddles page <http://mathriddles.williams.edu/> (with several students). The goal is to help students and teacher see that math can be both fun and useful. Eventually we plan on adding a student / teacher's corner, where we will have extensive discussions about how to think about / attack these riddles, as well as additional reading about the mathematics behind these problems. For example, one of my favorites is the chess problem posted here: <http://mathriddles.williams.edu/?p=57>. The solution (or perhaps its better to say a very good way to find the solution) to this riddle involves the same principle as airlines use to determine their optimal schedules! Building on these problems, I am working with OIT at Williams to create short web videos for general audiences. Two are online at

- <http://www.youtube.com/watch?v=Esa2TYwDmWA&feature=g-upl> and
- <http://www.youtube.com/watch?v=aMorr1h4Egs&feature=g-upl>.

7 VIGRE Undergraduate Mathematics Laboratory

7.1 Purpose

For many years I was involved in the design and implementation of a new advanced undergraduate class at Princeton, NYU, and Ohio State. Sponsored by a VIGRE grant from the NSF, Professor Sarnak and I created an Undergraduate Mathematics Laboratory at Princeton. We then brought the class to the Courant Institute, NYU (where we co-taught it with Alex Barnett), and I then brought the course to Ohio State. I have run a smaller version at Brown (by myself and with Jill Pipher in 2006), and then at Williams I taught a senior seminar from a book I wrote for the course.

The statements and simple cases of many interesting conjectures are accessible to undergraduates; further, there is often very little numerical support for the sweeping generalizations that are claimed in the conjectures. Undergraduates learn the necessary theory and quickly become involved in cutting edge research. They see what types of problems mathematicians study, and experience what it would be like to be a graduate student by doing original (guided) research.

While many graduate students and most postdocs have taught a standard class, few have mentored students or helped design research programs; additionally, many graduate students have yet to conduct original research, write papers or give research talks. Under the supervision of the faculty, these skills are built by having the graduate students help choose problems to investigate with

the undergraduates they are mentoring. The integrated nature of the program provides valuable training and exposure not seen in a typical class or research group.

7.2 Course Structure

The courses run roughly as follows: the professors and postdocs lecture on various topics for a few weeks. Participants often come from very diverse backgrounds with different skills, and supplemental lectures are given on needed background material. For example, to investigate certain conjectures we rarely need an entire course on Probability or Complex Analysis, but rather a few key results. Students are quickly given problems to think about / experiment with; as the semester progresses, the students and staff break into smaller groups. The general lectures are replaced with more specific presentations. At the end of the semester, students submit a .tex version of their work, documented computer codes, and present their research to the class.

When possible, we try to choose an integrated series of problems, so that students will be able to converse with each other about their work. We have worked hard to maintain good faculty to student ratios (usually about 1:3 or 1:4). For example, at Ohio State we had two senior faculty, two post-docs, and eight undergraduate and graduate students. One group was constructing families of elliptic curves with moderate to high rank, which was used by another group looking at the effect of additional zeros at the central point on the first zero above the central point, both of which help a third group investigating family dependent lower order corrections to distribution of zeros near the central point.

Typically these classes have a few faculty members, several graduate students or postdocs, and an undergraduate computer assistant. These are mathematics classes, not computer classes. While many undergraduates have programming experience, not much is required to do good investigations, due to the power of packages such as Mathematica, Maple, Matlab, Magma, C/PARI, etc. Not all the staff needs to be expert in these systems; typically only a subset is, and an undergraduate with strong skills is sometimes hired to be point man for debugging. Also, after running these courses for years, we have assembled an extensive database of documented programs, which makes it easy for future investigators with little computer experience to continue these researches.

7.3 Topics and Results

Projects from previous years are available on-line:

- Princeton: <http://www.math.princeton.edu/~mathlab/index.html>
- NYU: http://www.williams.edu/Mathematics/sjmiller/public_html/math/generalmath/uml@nyu
- AIM: http://www.williams.edu/Mathematics/sjmiller/public_html/math/generalmath/AIM
- Ohio State: http://www.williams.edu/Mathematics/sjmiller/public_html/math/generalmath/reu

Some previous topics of investigation include

1. Hardy-Littlewood Circle Method (Varieties; Goldbach; Germain Primes)
2. Random Matrix Theory (band matrices; Toeplitz and Palindromic Toeplitz matrices; truncated Cauchy matrices; sparse matrices)
3. Ramanujan Graphs / Random Graphs / k -Regular Graphs
4. Elliptic Curves (Birch and Swinnerton-Dyer Conjecture; excess rank, points of low height, signs of functional equations, first zero above the central point, spacings between zeros; Sato-Tate, constructing one-parameter families with rank)

5. Primality Testing
6. Equidistribution of Roots of Polynomials mod p
7. Continued Fractions (distribution of digits; special families; closed form expressions; periodic continued fractions)
8. Poissonian Behavior (especially of $\{n^k \alpha\}$)
9. Dynamical Piston
10. Lone Runner
11. Interval Exchanges
12. $3x + 1$ Problem
13. Benford's Law and Digit Bias

7.4 My Duties

My duties range from designing the class and choosing problems to giving background and advanced lectures to supervising undergraduate and graduate research and finally to mentoring graduate students and postdocs in how to advise and mentor undergraduates and design research programs. I am also responsible for teaching the participants how to use the software (C, PARI, Matlab, Maple, Mathematica, LaTeX), as well as providing help with programming and efficient algorithm design.

7.5 Summary

Designing and running the UML has shown me what it is like to supervise undergraduate and graduate research. What matters most is not the background of the participants, but their enthusiasm and interest in mathematics. There is no dearth of accessible problems, and in fact some of the best projects were done by students with the weakest background. With other professors, I supervised a variety of simultaneous research projects (eight students the first year, eleven the second, twenty in the third, eight in the fourth and three in the fifth). The UML gave me two rare opportunities for a graduate student: I was an organizer / coordinator for a research group, and I participated in the design and creation of a non-standard class. I also learned how to budget my time so that my own research does not suffer.

It has been a very exciting and enjoyable class to teach. I helped students investigate unsolved conjectures, and helped to train / work with graduate students and postdocs. While it is time consuming, the problems are exciting and worth doing, and at the end of the year I have helped prove new results (or at least gather experimental evidence in their support). It does take significantly longer to solve and write-up a project than it would if I worked alone, but I am exposed to a lot more mathematics and I help train future colleagues, benefiting from their enthusiasm and viewpoints.

Results from these researches have been presented at conferences and accepted in research journals for publication. Also, Ramin Takloo-Bighash and I have written a book, *An Invitation to Modern Number Theory* (Princeton University Press, 2006). The purpose of our book is to provide an introduction to current problems in Number Theory. While capable of being used as a standard text, it is based on lectures, problems and results from the past few years of the UML. Several new results in the fields are contained, as well as extensive literature review and lists of additional topics for research.

Finally, the course has evolved to more heavily include graduate students. Building on our success with undergraduates, we are expanding the role of graduate students. Instead of solely involving upper level graduate students as mentors, we now have pre-Generals graduate students,

who can explore an area of mathematics before having to choose a thesis topic. Both sets of graduate students are given assistance in mentoring and learning how to lecture on current research.