# PI MU EPSILON: NEW PROBLEMS

STEVEN J. MILLER (EDITOR)

## 1. Problems: Spring 2014

This department welcomes problems believed to be new and at a level appropriate for the readers of this journal. Old problems displaying novel and elegant methods of solution are also invited. Proposals should be accompanied by solutions if available and by any information that will assist the editor. An asterisk (*) preceding a problem number indicates that the proposer did not submit a solution.

Solutions and new problems should be sent to the Problem Section Editor Steven J. Miller, preferably by email to sjm1@williams.edu (or by snail mail to 18 Hoxsey St, Williams College, Williamstown), MA 01267; proposers of new problems are strongly encouraged to use LaTeX. Please submit each proposal and solution preferably typed or clearly written on a separate sheet, properly identified with your name, affiliation, email address, and if it is a solution clearly state the problem number. Solutions to problems in this issue should be mailed to arrive by XXXXXXXX. Solutions identified as by students are given preference.

**#1288:** *Proposed by Gabriel Prajitura, Mathematics Department, SUNY Brockport.*

A term $a_k$ of a sequence $\{a_n\}$ is called a **local extreme** if either $a_{k-1} \leq a_k \geq a_{k+1}$ or $a_{k-1} \geq a_k \leq a_{k+1}$. (a) If a sequence has infinitely many local extreme terms prove that the sequence is convergent if and only if the subsequence of all local extreme terms is convergent. (b) Show that Part (a) is no longer true if in the definition of a local extreme $\leq$ and $\geq$ are replaced by $<$ and $>$ respectively.

**#1289:** *Proposed by Mike Pinter, Belmont University, Nashville, TN.*
In honor of the centennial of Pi Mu Epsilon, solve in base 16

$$\begin{array}{r} PMEMATH \\ \underline{+\ \ SOCIETY} \\ HUNDRED \end{array}$$

(note there are 15 different letters).

*Date*: January 6, 2014.

**#1290:** *Proposed by Neculai Stanciu, George Emil Palade School, Buzău, Romania and Titu Zvonaru, Comănesti, Romania.*

Consider a set of five distinct positive real numbers such that if we take all products of pairs of these numbers then only seven distinct numbers are formed. Thus if the numbers are $0 < x_1 < x_2 < x_3 < x_4 < x_5$, if we look at the set formed from all products $x_i x_j$ with $i \neq j$ then there are only seven distinct numbers. Prove the $x_i$'s form an geometric progression; in other words, there is a $r$ such that $x_{i+1} = r x_i$ for $i \in \{1, 2, 3, 4\}$.

**#1291:** *Chirita Marcel, Bucharest, Romania.*

Given $x_1, x_2, x_3, x_4, x_5, x_6 \in (0, \infty)$ such that

$$\frac{1}{x_1 + x_2} + \frac{1}{x_3 + x_4} + \frac{1}{x_5 + x_6} \;=\; 1,$$

prove that

$$\left(\sum_{i=1}^{6} x_i\right)^2 \left(\sum_{i=1}^{6} x_i + 9\right) \;\geq\; 54(x_1 + x_2)(x_3 + x_4)(x_5 + x_6).$$

**#1292:** *Proposed by Moti Levy, Rehovot, Israel.*

Let $f(x)$ and $f^2(x)$ be Riemann-integrable functions on $[0, 1]$, and let $g(x)$ be a twice-differentiable function on $[0, 1]$ such that $g(0) = 1$.

a) Show that

$$\lim_{n\to\infty} \prod_{k=1}^{n} g\left(\frac{1}{n} f\left(\frac{k}{n}\right)\right) \;=\; \exp\left(g'(0) \int_0^1 f(x)\, dx\right).$$

b) Find a suitable choice of the functions $f(x)$ and $g(x)$ to solve **Problem 1892** from *Mathematics Magazine* (proposed by Jose Luis Dıaz-Barrero):

$$\lim_{n\to\infty} \frac{1}{n^n} \prod_{k=1}^{n} \frac{n\sqrt{n} + (n+1)\sqrt{k}}{\sqrt{n} + \sqrt{k}} \;=\; \frac{4}{e}.$$

**#1293:** *Proposed by Steven J. Miller, Williams College.*

The following is from the 2010 Green Chicken Contest between Middlebury and Williams.

Every year Middlebury and Williams have a math competition among their students, with the winning team getting to keep the infamous Green Chicken till the following year; see

http://web.williams.edu/Mathematics/sjmiller/public_html/greenchicken/index.htm

for pictures and additional history and problems. The following is a modification of a problem from 2010.

Instead of taking a math contest, Middlebury and Williams decide to settle who gets the Green Chicken by playing the following game. Consider the first one million positive integers. Player A's goal is to choose 10,000 of these numbers such that at the end of the choosing procedure there are at least 20 pairs of chosen integers with the same positive difference (for example, (12,39), (39,66) and (101,128) count as three pairs with a difference of 27). A turn consists of Player A choosing 10 numbers, and then Player B moving up to 10 of *any* number chosen to any unchosen number. We keep playing until A has chosen 10,000 numbers, allowing B to get its final turn. Determine which player has a winning strategy, and prove your claim.

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