PI MU EPSILON: NEW PROBLEMS

STEVEN J. MILLER (EDITOR)

1. Problems: Fall 2013

#1283: *Proposed by D. Andrica, E. Ionascu and R. Stephens, Columbus State University, Columbus, GA.*

Let k and n be positive integers. For the set $S_{k,n} = \{1^k, 2^k, \ldots, n^k\}$, consider the question "Can $S_{k,n}$ be partitioned into two nonempty subsets, each having the same sum?" Let $P_{k,n}$ be the number of ways to partition $S_{k,n}$ in this manner. For example:

- $P_{1,3} = 1$ with $S_{1,3} = \{1, 2\} \cup \{3\}$.
- $P_{1,4} = 1$ with $S_{1,4} = \{1, 4\} \cup \{2, 3\}$.
- $P_{1,5} = P_{1,6} = 0$.
- In fact, $P_{k,n} = 0$ whenever the sum of the members of $S_{k,n}$ is odd.
- $P_{1,7} = 4$ with $S_{1,7} = \{1, 2, 4, 7\} \cup \{3, 5, 6\}$, or $\{1, 6, 7\} \cup \{2, 3, 4, 5\}$, or $\{2, 5, 7\} \cup \{1, 3, 4, 6\}$, or $\{3, 4, 7\} \cup \{1, 2, 5, 6\}$.
- $P_{2,n} = 0$ for $n \leq 6$.
- $P_{2,7} = 1$ with $S_{2,7} = \{1^2, 2^2, 4^2, 7^2\} \cup \{3^2, 5^2, 6^2\}$.

Question 1: Find the smallest value of n for which $P_{2,n} > 1$. Explain your answer and identify any technology used.

Question 2: Find a general "formula" for determining P, the number of ways that the finite set $S = \{\theta_1, \theta_2, \ldots \theta_n\}$ of integers can be partitioned into two nonempty subsets, each having the same sum.

#1284: *Proposed by Heidi Burgiel, Department of Mathematics and Computer Science, Bridgewater State College, Bridgewater, MA 02325.*

Senior Janine Barros of Bridgewater State College wears earrings (pictured in Figure 1) that are made from the shell of a large seed pod with a cord threaded or "stitched" through it.

This design can be interpreted as a connected graph with 17 vertices, with each stitch forming an edge of the graph. The vertices of the graph are the points where the cord passes through the pod; on the reverse side of the pod is a graph with the same vertices. If we assume that the cord is tied into a loop, a stitch on the front between vertices a and b must

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FIGURE 1. A picture of the earing.

arise from and descend to stitches on the back, one ending at a and one at b.

Question: Is it possible that the graph on the back side of the pod is identical to that on the front? If so, can this solution be stitched using only a single loop of thread? What other pairs of graphs can be created by this stitching operation? What pairs of graphs *cannot* be stitched together in this way?

#1285: *Proposed by Tom Moore, Math and CS/Hart Hall, Bridgewater State University, Bridgewater, MA 02325*

Find all solutions in integers a and b to the equation $(a+3)(a^2+3) = b^2+7$.

#1286: *Proposed by Gabriel Prajitura, Mathematics Department, SUNY Brockport.*

Find an arithmetic progression of natural numbers such that the distance from any term of the progression to any square number is at least 7.

#1287: *Proposed by David Rhee, Massachusetts Institute of Technology, Boston, MA.*

Amy and Peter are sharing a cake. Amy will cut it into two pieces. Peter then cuts one of the pieces into two. This is followed by a second cut by Amy and a second cut by Peter, so that there will be five pieces, of sizes $0 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$, with $a_1 + a_2 + a_3 + a_4 + a_5 = 1$. Amy will get the three pieces of sizes a_1, a_3 and a_5, while Peter will get the remaining two pieces. What is the maximum amount of the cake Amy can get?

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