

MATH 209: HW DUE APRIL 22ND

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**Question 1 (40 points)** : Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix}.$$

Show the eigenvectors are  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ . Find the matrix  $S$  that diagonalizes  $A$ , and solve the first order linear non-homogenous system of differential equations

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = A \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \exp(-t) \\ \exp(3t) \end{pmatrix}.$$

**Question 2 (40 points)** : Find all solutions to the following (or prove no solution exists):

- (1)  $y'(x) + xy(x) = \exp(-x^2/2)$  with  $y(0) = 1$ .
- (2)  $(x^2 + 4x + 4)dy/dx = \exp(-y)$ .
- (3)  $y''(x) + (x^2 + 1)y'(x) + y(x) = 0$ .
- (4)  $y''(x) + 2y'(x) - 15y(x) = x^2 + x + 1$  with  $y(0) = 1$  and  $y'(0) = 1$ .

**Question 3 (20 points : extra credit)** : Let  $F_n$  be the  $n^{\text{th}}$  Fibonacci number ( $F_{n+2} = F_{n+1} + F_n$  with  $F_0 = 0$  and  $F_1 = 1$ ). Evaluate  $\sum_{n=0}^{\infty} F_n/3^n$  (or show the sum does not converge).