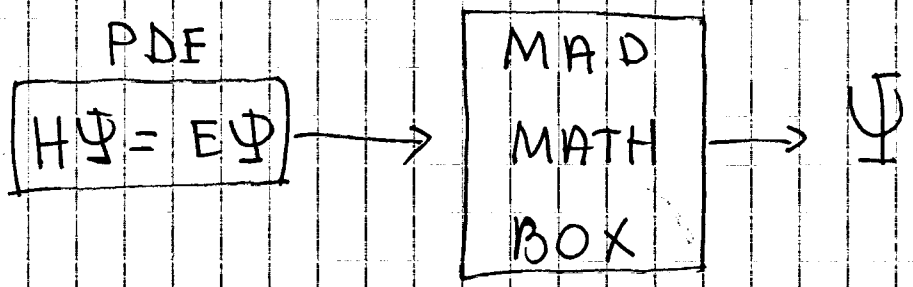
$$H\Psi = E\Psi$$

$$H = KE + V$$

$$KE = -\frac{\hbar^2}{2m}\nabla^2$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

V DEPENDS ON THE SYSTEM



HYDROGEN ATOM

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\underset{\sim}{r}) + V(r)\Psi(\underset{\sim}{r}) = E\,\Psi(\underset{\sim}{r})$$

$$\underset{\sim}{r} = (x, y, z) = (r, \theta, \varphi)$$

CARTESIAN SPHERICAL

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\Psi(r,\theta,\varphi) + V(r)\Psi(r,\theta,\varphi) = E\,\Psi(r,\theta,\varphi)}$$

$$V(r) = -\frac{Z|q_e|^2}{4\pi\epsilon_0}\frac{1}{r}$$

Sol

$$\Psi(r,\theta,\varphi) = \underbrace{R_{n,\ell}(r)}_{\text{RADIAL}}\,\underbrace{Y_\ell^m(\theta,\varphi)}_{\text{ANGULAR}}$$

$$\boxed{E_n = -\left(\frac{m|q_e|^4}{8\epsilon_0^2 h^2}\right)\frac{Z^2}{n^2}}$$

ANGULAR MOMENTUM $\leftrightarrow$ $\ell(\ell+1)\hbar^2$

PROJECTION OF ANGULAR MOMENTUM IN THE DIRECTION OF A MAGNETIC FIELD $\leftrightarrow m\hbar$

$$\Psi_{n\ell m}(r,\theta,\varphi) = R_{n\ell}(r)\, Y_{\ell}^{m}(\theta,\varphi)$$

n = PRINCIPLE QUANTUM NUMBER

ℓ = ANGULAR MOMENTUM L

m = PROJECTION OF $L \longrightarrow L_z$

$$L^2 \longrightarrow \ell(\ell+1)\hbar^2$$

$$L_z \longrightarrow \pm m\hbar$$

$R_{n\ell}(r)$ RADIAL WAVE FUNCTION

$Y_{\ell}^{m}(\theta,\varphi)$ ANGULAR PART.

RADIAL FUNCTION IS PARTICULAR OF THE HYDROGEN-LIKE POTENTIAL

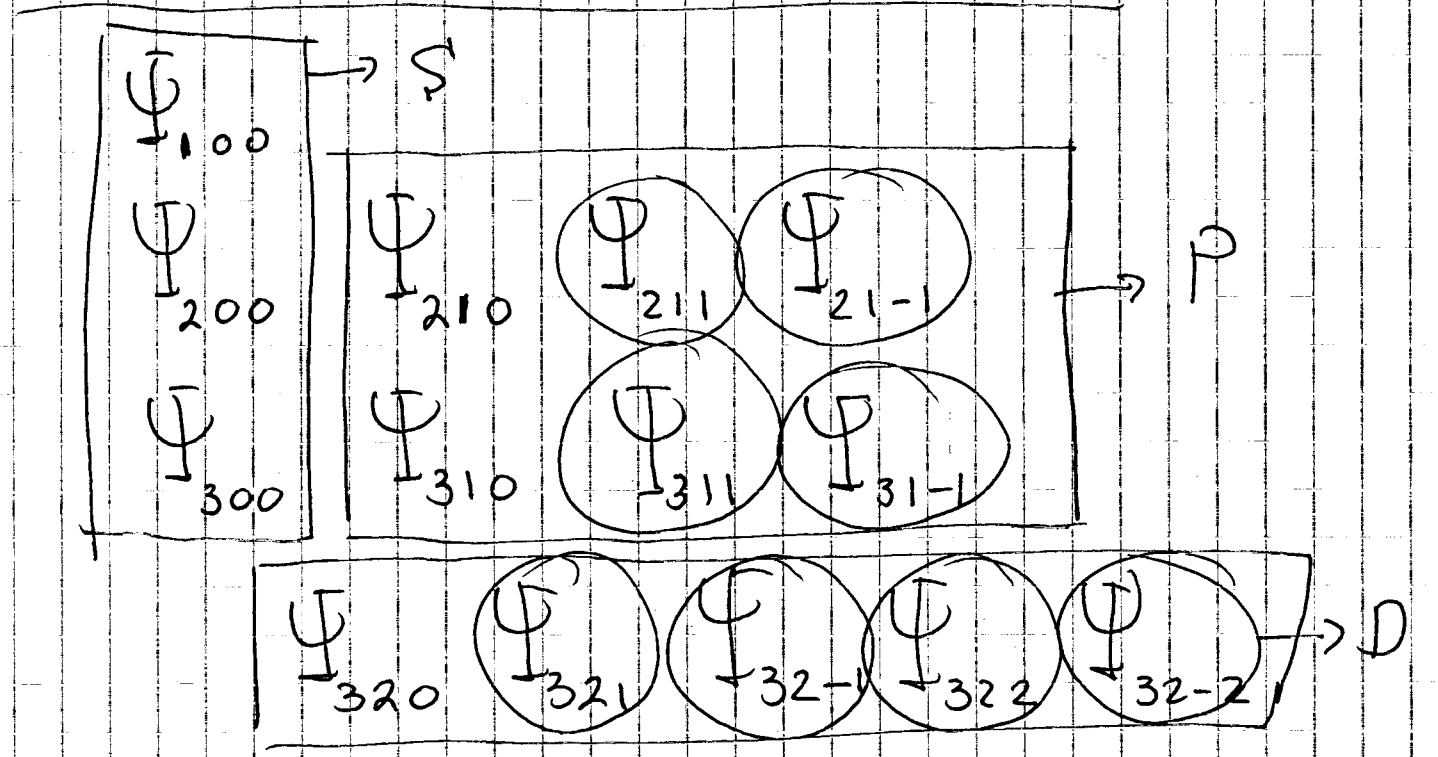
$$V = -\frac{Z|q_e|^2}{4\pi\varepsilon_0}\frac{1}{r}$$

Y_{ℓ}^{m} ONLY FEW ARE REAL FUNCTIONS

$n = 1, 2, 3, 4, \ldots$

$l = 0, 1, 2, \ldots, n-1$

$m = 0, \pm 1, \pm 2, \ldots, \pm l$



◯ → ARE NOT REAL FUNCTIONS

COMBINATION OF y_1^1 AND y_1^{-1} ⇒ p orbitals

COMBINATION OF y_2^1 AND y_2^{-1}, y_2^2 AND y_2^{-2} ⇒ d-orbitals