15.6 (a) Substituting in $c = \lambda\nu$ gives the frequency of 488 nm light to be 6.14×10^{14} s^{-1}.
(b) We divide twice the distance from the earth to the moon by the speed of light. The answer is 2.5 s.

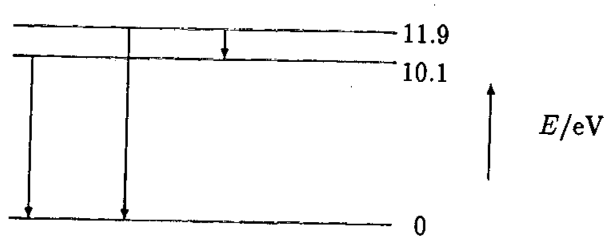
15.14 The wavelength of the radiation must be short enough to make the photon energetic enough to eject an electron from the surface of the tungsten. The work function is 7.29×10^{-19} J, which is supplied by photons of wavelength 272 nm or shorter (computed using $\lambda = hc/E$). If the ejected electron has a velocity of 2.00×10^6 m s^{-1}, its kinetic energy is 1.82×10^{-18} J, by application of the formula $K.E. = 1/2\, mv^2$ with $m = 9.109 \times 10^{-31}$ kg and v as given. The energy required from the photon is the work function plus this kinetic energy; the sum equals 2.55×10^{-18} J. The corresponding λ is 77.9 nm.

15.20 (a) The energy carried by a photon is the product of Planck's constant and its frequency $E = h\nu$. In this case $E = 3.8 \times 10^{-19}$ J.

(b) The power of the laser is 10 W, which is 10 J s^{-1}. Hence:

$$\frac{10 \text{ J}}{1 \text{ s}} \times \left(\frac{1 \text{ photon}}{3.82 \times 10^{-19} \text{ J}}\right) = 2.6 \times 10^{19} \frac{\text{photon}}{\text{s}}$$

15.22 The three emission lines connect each possible pair of levels (see diagram).



The corresponding wavelengths of emitted light are given by

$$\lambda = \frac{hc}{\Delta E} = \frac{12.3982 \times 10^{-7} \text{ m}}{V_{\text{thr}}[\text{V}]}$$

(recall Example 15.3). Substituting $V_{\text{thr}} = 10.1$, 11.9, and 1.8 V (the last of these is the voltage difference between the two excited states) gives wavelengths of

$$\lambda = 1.23 \times 10^{-7} \text{ m}, \qquad 1.04 \times 10^{-7} \text{ m}, \qquad 6.9 \times 10^{-7} \text{ m}$$

which can be written as 123, 104, and 690 nm.

15.24 (a) The energy change to remove an electron from a ground-state atom is

$$\Delta E = 13.6 \text{ eV} \left(\frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right) = 2.179 \times 10^{-18} \text{ J}$$

The wavelength of a photon supplying this amount of energy is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J s})(2.9979 \times 10^8 \text{ m s}^{-1})}{2.179 \times 10^{-18} \text{ J}} = 9.12 \times 10^{-8} \text{ m} = 91.2 \text{ nm}$$

(b) Solve the equation $\frac{1}{2}mv^2 = \Delta E$ for v and substitute

$$v = \sqrt{\frac{2\Delta E}{m}} = \sqrt{\frac{2(2.179 \times 10^{-18}\ \mathrm{J})}{9.109 \times 10^{-31}\ \mathrm{kg}}} = 2.19 \times 10^6\ \mathrm{m\ s^{-1}}$$

Converting to miles per hour gives

$$v = 2.19 \times 10^6\ \mathrm{m\ s^{-1}} \left(\frac{1\ \mathrm{ft}}{0.3048\ \mathrm{m}}\right)\left(\frac{1\ \mathrm{mi}}{5280\ \mathrm{ft}}\right)\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = 4.89 \times 10^6\ \mathrm{mi\ h^{-1}}$$

(c) For thermal excitation, $k_B T \approx \Delta E$

$$T \approx \frac{\Delta E}{k_B} = \frac{2.179 \times 10^{-18}\ \mathrm{J}}{1.381 \times 10^{-23}\ \mathrm{J\ K^{-1}}} = 1.58 \times 10^5\ \mathrm{K}$$

15.26 According to the Bohr model, the radius of a one-electron atom or ion is

$$r = \frac{n^2}{Z}a_o = \frac{n^2}{Z}(5.29 \times 10^{-11}\ \mathrm{m})$$

Substitution of $Z = 2$ for helium and $n = 5$ gives $r = 6.61 \times 10^{-10}$ m. The energy of any state of a one-electron atom or ion is given by

$$E = -\frac{Z^2}{n^2}(2.18 \times 10^{-18}\ \mathrm{J})$$

In the case of a He^+ ion in the $n = 5$ state, this energy equals -3.49×10^{-19} J. Removing the electron means changing the energy of the atom to $E = 0$. The change in energy of one atom is this final value minus the initial value, or $+3.49 \times 10^{-19}$ J. For a mole of atoms the energy change is Avogadro's number times larger or 210 kJ. The energy of the He^+ ion in the $n = 3$ state is -9.69×10^{-19} J. The change in energy of the ion in the $5 \rightarrow 3$ transition is the $n = 3$ energy (the final energy) minus the $n = 5$ energy (the initial energy). Hence $\Delta E = -6.20 \times 10^{-19}$ J. The transition gives off energy, as shown by its negative ΔE. The frequency of the photon that carries this energy away is $9.36 \times 10^{14}\ \mathrm{s^{-1}}$, and the wavelength is 320 nm.

15.32 (a) We know the mass of the electron so we can calculate the v of the electrons from their kinetic energy, from the relationship $K.E. = 1/2\, m_e v^2$. It is $6.614 \times 10^6\ \mathrm{m\ s^{-1}}$. The wavelength is 0.110 nm (using $\lambda = h/p = h/m_e v$).

(b) The helium atom moving at 353 $\mathrm{m\ s^{-1}}$ gives $\lambda = 0.282$ nm.

(c) The krypton atom moving at 299 $\mathrm{m\ s^{-1}}$ has $\lambda = 0.0159$ nm.

15.36

$$\Delta E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \mathrm{J\ s})(2.9979 \times 10^8\ \mathrm{m\ s^{-1}})}{800 \times 10^{-9}\ \mathrm{m}} = 2.483 \times 10^{-19}\ \mathrm{J}$$

$$\Delta E = \frac{h^2}{8\, m_e L^2}\left[(2)^2 + (1)^2 + (1)^2 - (1)^2 - (1)^2 - (1)^2\right] = \frac{h^2}{8\, m_e L^2}[3]$$

$$L = \sqrt{\frac{(6.626 \times 10^{-34}\ \mathrm{J\ s})^2\ [3]}{8(9.109 \times 10^{-31}\ \mathrm{kg})(2.483 \times 10^{-19}\ \mathrm{J})}} = 8.53 \times 10^{-10}\ \mathrm{m} = 8.53\ \text{Å}$$

PROBLEM 15.69

$$\lambda_{max} = 0.20 \frac{hc}{k_B T} = \frac{hc}{5 k_B T}$$

$$\lambda_{max} = 465\ nm$$

$$\boxed{T = \frac{hc}{5 \lambda_{max} k_B}}$$

$$= \frac{6.626 \times 10^{-34}\ \cancel{J}\cancel{s}\ 2.9979 \times 10^{8}\ \cancel{m}\ \cancel{s^{-1}}}{5(1.3066 \times 10^{-23}\ \cancel{J}\ K^{-1})\ 465 \times 10^{-9}\ \cancel{m}}$$

$$= \frac{6.626 \times 2.9979}{5(1.3066)(465)} \frac{10^{-26}}{10^{-32}} K$$

$$= 6.54 \times 10^{3} K$$

15.86 (a) The probability density of finding an electron in the vicinity of any point is equal to the square of the wave function of the electron evaluated at that point. Thus, the probability density of finding the 1*s* electron of the H atom at a distance r from the nucleus is

$$\psi^2 = (1/\pi a_0^3)\exp(-2r/a_0)$$

The portion in the first parentheses equals 2.15×10^{30} m^{-3} and the exponential term is unitless. At r equals 0 the exponential term equals 1 and ψ^2 is 2.15×10^{30} m^{-3}.

The probability of finding the electron *exactly at* a mathematical point is zero because a point has no volume to accommodate the electron. The small sphere centered at the nucleus has however a volume of 1 pm^3 (1.0×10^{-36} m^3). Over the very short distance between the center and surface of this sphere ψ^2 stays nearly constant at 2.15×10^{30} m^{-3}. It follows that the probability of finding the electron within the small sphere is about:

$$p = 2.15 \times 10^{30} \text{ m}^{-3} \times 1.00 \times 10^{-36} \text{ m}^3 = 2.15 \times 10^{-6}$$

(b) At a distance a_0 or 52.9 pm (0.529×10^{-10} m) from the nucleus, the value of the functions ψ^2 is *less* than it is at the nucleus. The exponential part of the function drops rapidly as r increases:

$$\psi^2(\text{at } a_0) = (2.15 \times 10^{30} \text{ m}^{-3})e^{-2r/a_0} = 2.91 \times 10^{29} \text{ m}^{-3}$$

Assume that ψ^2 is constant throughout the 1 pm^3 volume which the problem specifies. The chance of finding the electron at 52.9 pm in a fixed direction is

$$p = 2.91 \times 10^{29} \text{ m}^{-3} \times 1.0 \times 10^{-36} \text{ m}^3 = 2.91 \times 10^{-7}$$

(c) A spherical shell of thickness 1 pm and radius 52.9 pm has a volume:

$$V_{shell} = 4\pi r^2 \Delta r = 4\pi(52.9 \text{ pm})^2 \times 1 \text{ pm} = 3.52 \times 10^{-32} \text{ m}^3$$

This substantially larger volume naturally has a greater probability of holding the electron than does the tiny 1 pm^3 volume element considered in part b:

$$p = 2.91 \times 10^{29} \text{ m}^{-3} \times 3.52 \times 10^{-32} \text{ m}^3 = 0.0102$$