PROBLEM $\boxed{9}$ PS#4

THE UNCERTAINTY PRINCIPLE AND THE PARTICLE IN THE BOX

(30) a) THE UNCERTAINTY IN POSITION IS DEFINED AS:

$$\Delta X \equiv \sqrt{\langle x^2\rangle - \langle x\rangle^2},$$

WHERE

$$\langle x^m\rangle \equiv \int_0^L \psi_n^*(x)\, x^m\, \psi_n(x)\,dx$$

CALCULATE $\langle x^2\rangle$, $\langle x\rangle$ AND Δx FOR THE PARTICLE IN A BOX FOR ANY n.

(30) b) THE UNCERTAINTY IN MOMENTUM IS DEFINED AS:

$$\Delta P \equiv \sqrt{\langle p^2\rangle - \langle p\rangle^2},$$

WHERE

$$\langle p^m \rangle \equiv \int_0^L \psi_n^*(x) \left(p^m \psi_n(x) \right) dx .$$

IN Q.M.

$$p = -i\hbar \frac{d}{dx}$$

THUS

$$\langle p \rangle = \int_0^L \psi_m^*(x) \left(-i\hbar \frac{d\psi_m(x)}{dx} \right) dx$$

$$\langle p^2 \rangle = \int_0^L \psi_m^*(x) \left(-\hbar^2 \frac{d^2\psi_m(x)}{dx^2} \right) dx$$

CALCULATE $\langle p \rangle$, $\langle p^2 \rangle$ AND Δp FOR ANY VALUE OF n.

(40) c) VERIFY THE UNCERTAINTY PRINCIPLE

$$\Delta x \, \Delta p \geq \frac{\hbar}{2} = \frac{h}{4\pi}$$