```
DateList[]
```

```
{2009, 10, 8, 7, 16, 4.764546}
```

From problem 14 W.5, the function

$$\psi[\mathrm{x}] = \sqrt{\frac{105}{a^7}}\ x^2\,(x-a)^2$$

is not normalized. We can easily see that the normalization constant does not correspond to the function since:

$$\int_0^a x^4\,(x-a)^4\,dx$$

$$\frac{a^9}{630}$$

but

$$\int_0^a x^4\,(x-a)^2\,dx$$

$$\frac{a^7}{105}$$

So we have to options, either we change the power of the term (x-a) or we change the normalization constant.

Definitions:

$$\psi 1[x\_] := \sqrt{\frac{105}{a^7}}\ x^2\,(x-a)$$

$$\psi 2[x\_] := \sqrt{\frac{630}{a^9}}\ x^2\,(x-a)^2$$

Now we have two normalized functions

$$\int_0^a \psi 1[x]\,\psi 1[x]\,dx$$

$$\int_0^a \psi 2[x]\,\psi 2[x]\,dx$$

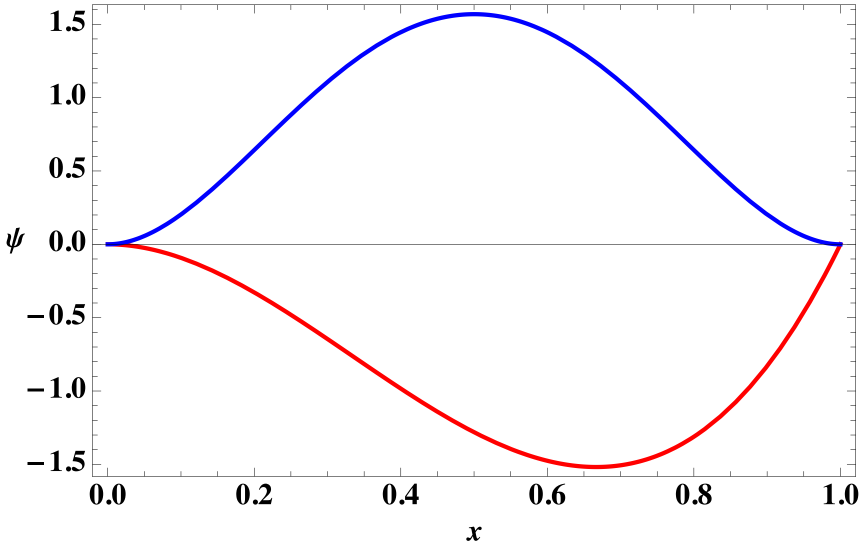
```
1

1
```

For completeness we plot the functions and the square of the functions, assuming that a=1.

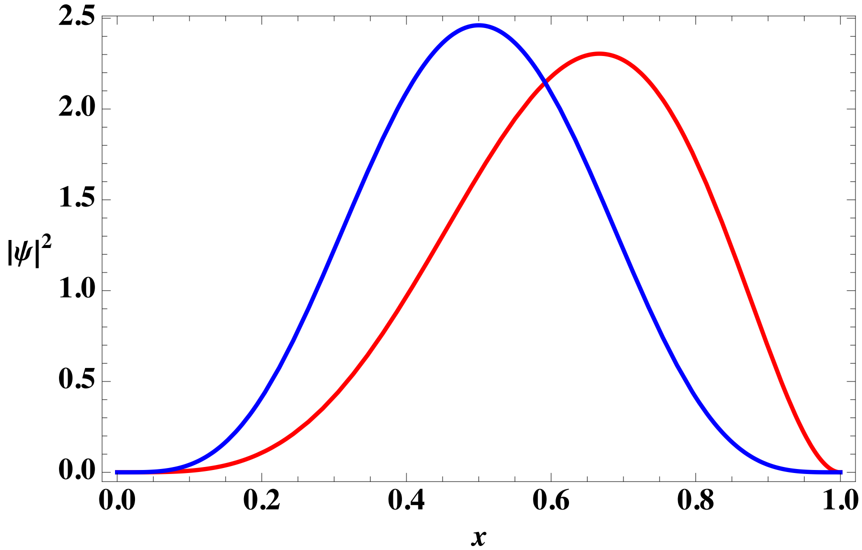
In[84]:=
```
Plot[{ψ1[x] /. {a → 1}, ψ2[x] /. {a → 1}}, {x, 0, 1}, PlotStyle → {{Thick, Red}, {Thick, Blue}},
 BaseStyle → {Bold, 14}, Frame → True, FrameLabel → {x, ψ}, RotateLabel → False, ImageSize → 400]
```

Out[84]=



In[85]:=
```
Plot[{ψ1[x]^2 /. {a → 1}, ψ2[x]^2 /. {a → 1}}, {x, 0, 1},
 PlotStyle → {{Thick, Red}, {Thick, Blue}}, BaseStyle → {Bold, 14},
 Frame → True, FrameLabel → {x, "|ψ|^2"}, RotateLabel → False, ImageSize → 400]
```

Out[85]=



To find the expansion coefficients and the probabilities, we first need to define the basis set:

$$\phi[m\_][x\_] := \sqrt{\frac{2}{a}}\ \mathrm{Sin}\left[\frac{m\,\pi\,x}{a}\right]$$

Next we proceed to define the coefficients for both functions $\psi 1$ and $\psi 2$.

For $\psi 1$ we have

```
c[m_] = Simplify[∫_0^a ϕ[m][x] ψ1[x] ⅆx, Assumptions → {m ∈ Integers}]
```

$$\frac{2\sqrt{210}\,\left(1+2\,(-1)^m\right)\sqrt{\frac{1}{a^7}}}{\left(\frac{1}{a}\right)^{7/2} m^3\,\pi^3}$$

Now we calculate the probabilities

```
prob1 = Table[{n, c[n]^2} // N, {n, 1, 10}];
```

and show them in a tabular form

```
TableForm[prob1, TableHeadings →
  {{"n =", "n =", "n =", "n =", "n =", "n =", "n =", "n =", "n =", "n ="}, {n, Prob1}}]
```

	n	Prob1
n =	1.	0.873736
n =	2.	0.122869
n =	3.	0.00119854
n =	4.	0.00191983
n =	5.	0.0000559191
n =	6.	0.000168545
n =	7.	7.42663×10^{-6}
n =	8.	0.0000299973
n =	9.	1.64409×10^{-6}
n =	10.	7.86362×10^{-6}

Finally we repeat the calculation for $\psi 2$

```
b[m_] = Simplify[∫_0^a ϕ[m][x] ψ2[x] ⅆx, Assumptions → {m ∈ Integers}]
```

$$\frac{12\sqrt{35}\,\left(-1+(-1)^m\right)\sqrt{\frac{1}{a^9}}\,\left(-12+m^2\,\pi^2\right)}{\left(\frac{1}{a}\right)^{9/2} m^5\,\pi^5}$$

Notice that for m even, we get a zero value of the coefficient

```
prob2 = Table[{n, b[n]^2} // N, {n, 1, 10}];
```

```
TableForm[prob2, TableHeadings →
  {{"n =", "n =", "n =", "n =", "n =", "n =", "n =", "n =", "n =", "n ="}, {n, Prob2}}]
```

	n	Prob2
n =	1.	0.97704
n =	2.	0.
n =	3.	0.0215179
n =	4.	0.
n =	5.	0.00121469
n =	6.	0.
n =	7.	0.000169503
n =	8.	0.
n =	9.	0.0000382824
n =	10.	0.

We observe quite a different behavior.