QUANTUM THEORY OF LIGHT

PHOTONS $\longrightarrow$ EXPLANATION FOR SOME EXPERIMENTS
photo electric effect

E&M WAVES $\longrightarrow$ EXPLANATION FOR SOME EXPERIMENTS
dispersion of white light

THE TRUE NATURE OF LIGHT IS NOT LONGER MEANINGFULL

FROM SPECIAL THEORY OF RELATIVITY

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

FOR A PHOTON

$$m_0 = 0$$

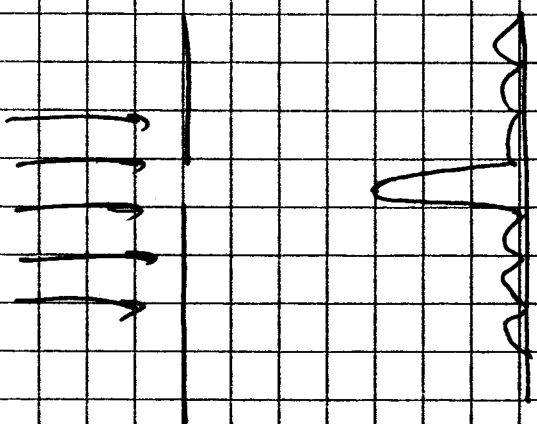
$$E = pc = h\nu$$

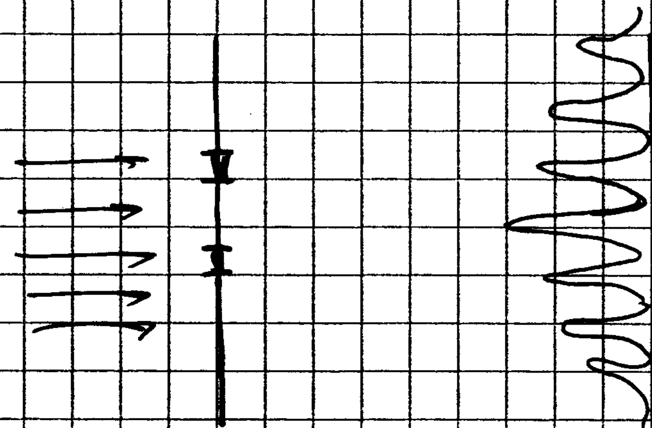
$$= p\nu\lambda = h\nu$$

$$\boxed{p = \frac{h}{\lambda}}$$

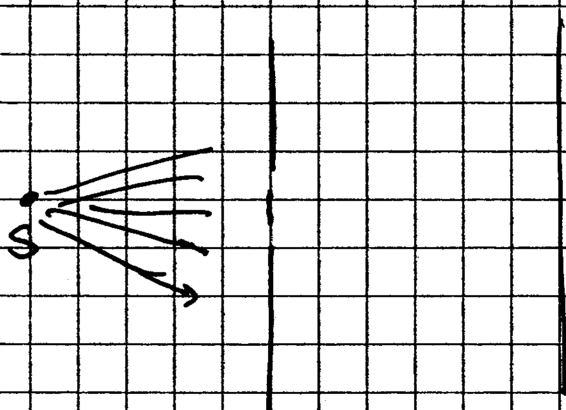
MASSLESS PHOTONS CARRY MOMENTUM!

LIGHT





ELECTRONS



POLY CRAYSTALINE FOIL

IF ONE DECREASES THE INTENSITY OF THE WAVE, THE # OF IMPACTS DECREASES PROPORTIONALLY. IN THE LIMIT OF LOW INTENSITY, ONE OBSERBS JUST SINGLE IMPACTS, EITHER ON THE CENTRAL PEAK OR ON ONE OF THE INTERFERENCE RINGS

AS WE MENTION EARLIER, THE NATURE OF LIGHT WAS NOT COMPLETELY UNDERSTOOD. IN MANY EXPERIMENTS LIGHT SHOWS A DEFINITE WAVE LIKE BEHAVIOR i.e., DISPERSION OF WHITE LIGHT INTO ITS SPECTRUM BY A PRISM, BUT THERE ARE MANY OTHER EXPERIMENTS IN WHICH LIGHT SEEMS TO BEHAVE AS A STREAM OF PHOTONS, i.e, PHOTO ELECTRIC EFFECT. THIS BEHAVIOR IS REFERRED AS THE WAVE-PARTICLE DUALITY OF LIGHT.

EINSTEIN SHOWED FROM RELATIVITY THAT THE MOMENTUM OF A PHOTON IS

$$p = \frac{h}{\lambda}$$

IF LIGHT CAN DISPLAY THIS WAVE-PARTICLE DUALITY, MATTER, WHICH CERTAINLY APPEARS TO BE PARTICLE-LIKE, MIGHT ALSO DISPLAY WAVE-LIKE PROPERTIES UNDER CERTAIN CONDITIONS.

THIS RATHER STRANGE PROPOSAL WAS MADE BY LOUIS DE BROGLIE IN 1924. IN THIS CASE ONE CAN ASSIGN A WAVELENGTH TO A PARTICLE OF MOMENTUM

$$p = mv$$

as

$$\lambda = \frac{h}{mv} = \frac{h}{p}.$$

DE BROGLIE RECOGNIZED THE SIMILARITY BETWEEN FERMAT'S PRINCIPLE OF LEAST TIME, WHICH GOVERNS THE PROPAGATION OF LIGHT, AND MAUPERTUIS' PRINCIPLE OF LEAST ACTION, WHICH GOVERNS THE PROPAGATION OF PARTICLES.

FROM BOHR'S ASSUMPTION

$$l = n\frac{h}{2\pi} = mvr$$

WE GET

$$2\pi r = n\frac{h}{mv} = n\lambda$$

THE WAVE LENGTH (λ) IS AN MULTIPLE OF THE CIRCUMFERENCE OF THE ORBIT.

	PARTICLE-LIKE	WAVE-LIKE
MATTER PARTICLES	✓ p	?
WAVES E&M	PHOTONS $p = \frac{h}{\lambda}$	λ

LOUIS de BROGLIE (1924)

FOR A PARTICLE WITH MOMENTUM p, WE ASSOCIATE A WAVELENGTH

$$\lambda = \frac{h}{p}$$

<u>EXAMPLE</u> a) 5 oz BASEBALL AT 90 mph

$$\lambda = 1.2 \times 10^{-24}\ \text{Å}$$
$$= 1.2 \times 10^{-25}\ \text{nm}$$

b) e^- AT 1.0 % OF c

$$\lambda = 2.43\ \text{Å} = 0.243\ \text{nm}$$

THEREFORE e^- SHOULD DIFFRACT

R. P. THOMSON (SON) PROVED EXPERIMENTALLY THAT e^- BEHAVES AS A WAVE (1937) NOBEL

J.J. THOMSON DISCOVERED THAT e^- IS A PARTICLE (1906) NOBEL

COMPTON EFFECT (1924)

SCATTERING OF X-RAYS BY FREE OR WEAKLY BOUND ELECTRONS.

STERN-GERLACH (1925)

PARAMAGNETIC ATOMIC BEAMS IN AN INHOMOGENEOUS MAGNETIC FIELD → QUANTIZATION OF ORIENTATION.

ATOM SITUATED IN AN EXTERNAL FIELD POSSES A PREFERED DIRECTION. THE ORIENTATION OF THE ATOMIC SYSTEM IS NOT ARBITRARY, BUT IT IS LIMITED TO CERTAIN DISCRETE VALUES.

IN 1925 DAVISSON AND GERMER WERE ABLE TO PERFORM AN EXPERIMENT THAT DEMOSTRATE THE DIFFRACTION OF ELECTRONS IN QUANTITATIVE AGREEMENT WITH A WAVELENGTH CALCULATED FROM deBROGLIE RELATION.

IN 1927 G.P. THOMSON OBSERVED THE DIFFRACTION OF ELECTRONS BY A THIN FILM OF CELLULOID.

THESE EXPERIMENTS EXTEND THE WAVE-~~LIGHT~~ PARTICLE DUALITY OF LIGHT TO MATTER.

<u>CORRESPONDANCE PRINCIPLE</u>

QT MUST APPROACH CT ASYMPTOTICALLY IN THE LIMIT OF LARGE QUANTUM NUMBERS

WE CAN ESTIMATE THE CHANGE

$$\Delta p \simeq p_{PHOTON} = \frac{h}{\lambda}.$$

THE MEASUREMENT INTRODUCES AN UNCERTAINTY IN THE MOMENTUM OF THE ELECTRON.

SINCE WE ARE USING LIGHT, THE POSITION OF THE e^- IS DETERMINED WITH AN UNCERTAINTY EQUAL TO λ

$$\Delta x \simeq \lambda.$$

HENCE

$$\Delta p \, \Delta x \simeq h$$

AT ANY INSTANT, A MEASUREMENT INTRODUCES THE INDETERMINACY.

THIS PRINCIPLE APPLIES TO SIMULTANEOUS MEASUREMENTS

<u>HEISENBERG UNCERTAINTY PRINCIPLE</u> (1926)

THE e^- CAN HAVE DEFINITE POSITION & MOMENTUM AT ANY INSTANT, BUT A MEASUREMENT INTRODUCES AN UNCERTAINTY.

$$\Delta p \Delta x \geq h$$

AN ELECTRON (QUANTUM MECHANICAL PARTICLE) CAN HAVE A TRAJECTORY. BUT WHEN WE TRY TO MEASURE POSITION AND MOMENTUM WE DESTROY IT.

IN A CHAMBER OF SATURATED WATER VAPOR, THE TRAJECTORY IS GIVEN BY A TRACK FORMED BY CONDENSED DROPPLETS. BUT SIZE IS TOO LARGE COMPARED WITH THE ATOMIC DIMENSIONS.

IN CLASSICAL MECHANICS, BOTH THE MOMENTUM AND POSITION ARE KNOWN AT THE SAME TIME. WITH THE USE OF NEWTON'S LAWS ONE CAN PREDICT THE FUTURE OF THE PARTICLE.

IN Q.M WE HAVE LESS INFORMATION THUS WE CANNOT PREDICT THE FUTURE OF THE PARTICLE. AT MOST WE WILL BE ABLE TO DETERMINE PROBABILITIES.

THEREFORE THE CLASSICAL IDEA OF A TRAJECTORY OF A PARTICLE IS INCOMPATIBLE WITH Q.M. FOR IT IS NECESSARY TO DEFINE AT EACH TIME THE POSITION AND THE MOMENTUM OF THE PARTICLE

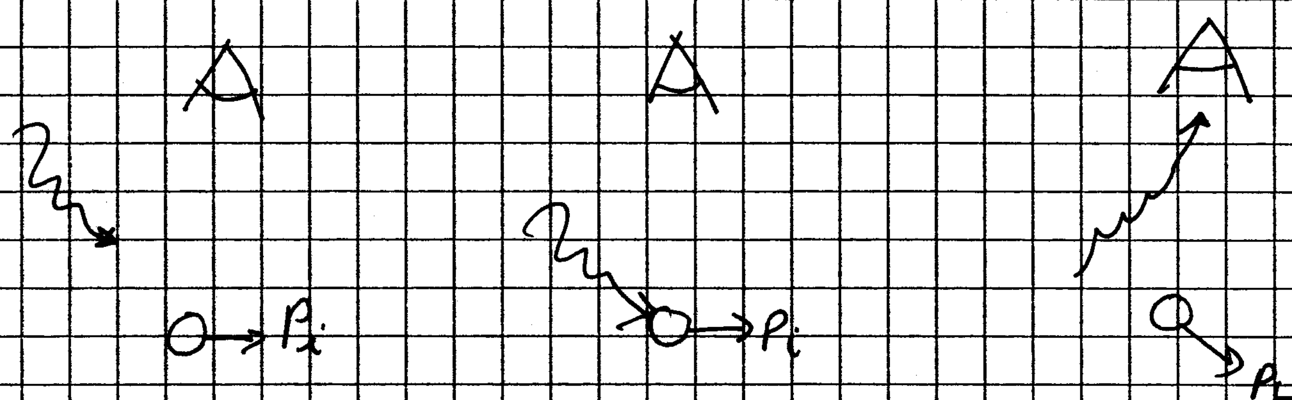
WHAT IS WRONG WITH BOHR'S MODEL?
A CLASSICAL TRAJECTORY IS ASSUME WITH POSITION AND MOMENTUM KNOWN WITH INFINITELY ACCURACY.

SINCE WE HAVE THE PARTICLE - WAVE DUALITY FOR MATTER, WHAT IS THE WAVE EQ. FOR MATTER WAVES?

WE NEED AN EQ. THAT INCLUDES
THE HEISENBERG PRINCIPLE AND
de BROGLIE PRINCIPLE

UNCERTAINTY PRINCIPLE

LET US CONSIDER THE MEASUREMENT OF THE POSITION AND MOMENTUM, AT THE SAME TIME, OF AN ELECTRON. IN THIS CASE WE HAVE TO INTERACT WITH THE e^- IN SOME WAY. FOR EXAMPLE CONSIDER LIGHT OF λ. THE PHOTONS INTERACT WITH e^- VIA COLLISIONS



MEASUREMENT $\leftrightarrow$ INTERACTION WITH THE SYSTEM

CLASSICAL THEORIES $\longleftrightarrow$ INTERACTIONS DOES NOT AFFECTS THE SYSTEM

QUANTUM THEORY $\longleftrightarrow$ INTERACTION AFFECTS THE SYSTEM

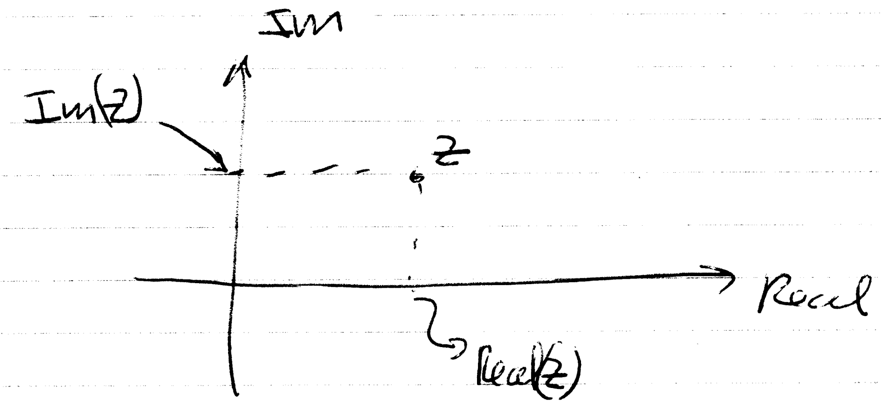
THE EXACT CHANGE IN MOMENTUM CAN NOT BE PREDICTED

COMPLEX NUMBERS

IF WE CONSIDER ONLY REAL NUMBER, WE HAVE NO SOLUTION TO THE FOLLOWING EQUATION:

$$x^2 + 1 = 0.$$

THEREFORE WE EXPAND THE REAL NUMBERS BY ADDING A COMPLEX AXIS OR IMAGINARY AXIS,



WHERE

$$i^2 = -1.$$

NOW COMPLEX NUMBERS HAVE TWO COMPONENTS: Real part AND Imaginary part.

$$z = (\mathrm{Real}(z), \mathrm{Im}(z))$$

AS A TWO DIMENSIONAL VECTOR

THE MAGNITUD OF Z, $|Z|$, IS DEFINED AS:

$$|Z| = \sqrt{[\mathrm{Real}(z)]^2 + [\mathrm{Im}(z)]^2}.$$

WE CAN ALSO REPRESENT Z AS:

$$\boxed{Z = \mathrm{Real}(z) + i\,\mathrm{Im}(z).}$$

IN THIS CASE WE RECALL THAT

$$x^2 - y^2 = (x+y)(x-y).$$

THEREFORE

$$\begin{aligned}
[\mathrm{Real}(z)]^2 + [\mathrm{Im}(z)]^2 &= [\mathrm{Real}(z)]^2 - i^2[\mathrm{Im}(z)]^2 \\
&= [\mathrm{Real}(z) + i\,\mathrm{Im}(z)][\mathrm{Real}(z) - i\,\mathrm{Im}(z)] \\
&= Z\, Z^*,
\end{aligned}$$

WHERE WE DEFINE THE COMPLEX CONJUGATE AS:

$$\boxed{Z^* = \mathrm{Real}(z) - i\,\mathrm{Im}(z)}$$

IN GENERAL IF $z \in \mathbb{C}$, WE GET THE COMPLEX CONJUGATE z^* BY CHANGING "i" by "$-i$."

USING THE COMPLEX CONJUGATE

$$|z| = \sqrt{z z^*}$$

OR

$$\boxed{|z|^2 = z z^*}$$

THE MAGNITUD SQURE OF $z \in \mathbb{C}$ IS EQUAL TO THE PRODUCT OF z TIMES ITS COMPLEX CONJUGATE