Boltzmann Distribution

CLASSICAL	QUANTUM
CONTINUUM OF ENERGY	DISCRETE ENERGY
LOW PRESSURE GAS	LOW PRESSURE GAS
KINETIC THEORY	ATOMIC SPECTRA
KINETIC ENERGY	ATOMIC ENERGY

RELATIVE POPULATION

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{-(E_i - E_j)/kT}$$

$k_B T$ AVERAGE ENERGY EXCHANGE DUE TO COLLISIONS

$$n_i \sim g_i e^{-E_0/k_B T}$$

a) $E_i - E_j \ll k_B T$ CLASSICAL

b) $E_i - E_j \gg k_B T$ QUANTUM

REVIEW

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \quad \text{WAVE EQUATION}$$

ANZATZ

$$u(x,t) = T(t)\,X(x)$$

$$\Rightarrow \quad \frac{1}{v^2}\frac{1}{T}\frac{d^2T}{dt^2} = \frac{1}{X}\frac{d^2X}{dx^2} = k$$

SPACE

$$\frac{d^2X}{dx^2} = kX$$

SOL

$$\sin(\alpha x)$$

$$\frac{d^2}{dx^2}\sin(\alpha x) = -\alpha^2\sin(\alpha x)$$

$$X(x) = a\sin(\alpha x)$$

B.C.

$$X(0) = 0$$

$$X(l) = 0$$

$$0 = a \sin(\alpha l) = 0$$

$$\Rightarrow \quad \alpha l = n\pi \qquad n \in \mathbb{N}$$

$$\boxed{\alpha = \frac{\pi}{l} n}$$

ANY OTHER SOLUTION DOES NOT SATISFY THE BOUNDARY CONDITIONS

$$\boxed{X(x) = a \sin\left(\frac{\pi}{l} n x\right)}$$

$$k = -\alpha^2$$

$$\frac{d^2 T}{dt^2} = -v^2 \alpha^2 T$$

SOL: $\sin(\alpha v t)$ OR $\cos(\alpha v t)$

I.C. $\quad u(x,0) = T(0)\, X(x) = 0$

$$\Rightarrow \quad T(t) \propto \sin(\alpha v t)$$

$$U_n = A_n \sin\left(\frac{\pi}{L} n v t\right) \sin\left(\frac{\pi}{L} n x\right)$$

HARMONIC

$U(x,t)$

ANY (STANDING) WAVE CAN BE BE EXPRESSED A A LINEAR COMBINATION OF THE HARMONICS (NORMAL MODES)

IF $U(x,0) = 0$ THEN

$$U(x,t) = \sum_{n=1}^{\infty} U_n \sin\left(\frac{\pi}{L} n v t\right) \sin\left(\frac{\pi}{L} n x\right)$$

$$\frac{d X}{dx^2} = k X \qquad (1)$$

B.C. $X(0) = 0 = X(L)$

POSSIBLE SOL.

$$e^{\sqrt{k}x} \qquad e^{-\sqrt{k}x}$$

$$\cos(\sqrt{k}x) \qquad \sin(\sqrt{k}x) \qquad e^{\pm i\sqrt{k}x}$$

BC $\Rightarrow \sin(\sqrt{k}x)$

$$\frac{d^2 \sin(\sqrt{k}x)}{dx^2} = -k \sin(\sqrt{k}x)$$

THEREFORE EQ(1) REQUIRES THAT EQ(1) BE MODIFIED TO:

$$\frac{d^2 X}{dx^2} = -\alpha^2 X$$

Sol $\sin(\alpha x)$

$\sin(\alpha L) = 0 \Rightarrow \alpha L = n\pi \quad \alpha = \frac{n\pi}{L}$