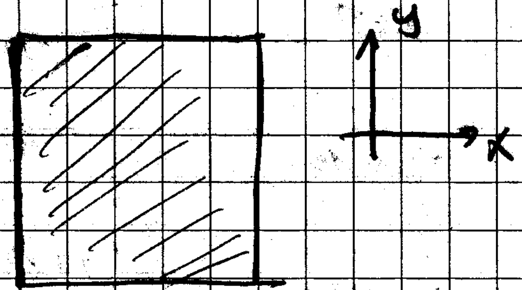
SQUARE DRUM



WAVE EQ

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

$\boxed{\text{B.C.}}$

$$u(0,y,t)=0$$
$$u(\ell,y,t)=0$$
$$u(x,0,t)=0$$
$$u(x,\ell,t)=0$$

ANZATZ

$$u(x,y,t)=T(t)\,F(x,y)$$

$$\frac{1}{F}\left[\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}\right]=\frac{1}{v^2}\frac{1}{T}\frac{d^2 T}{dt^2}=\text{CONST}$$

$$\text{CONST} \equiv -\beta^2$$

$$\frac{d^2 T}{dt^2} = -\beta^2 v^2 T$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = -\beta^2 F$$

$$F(x,y) = X(x)\, Y(y)$$

$$Y \frac{d^2 X}{dx^2} + X \frac{d^2 Y}{dy^2} = -\beta^2 X Y$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} = -\beta^2 = -p^2 - q^2$$

$$\frac{d^2 X}{dx^2} = -p^2 X$$

$$\frac{d^2 Y}{dy^2} = -q^2 Y$$

X and Y $\sim$ sin and cos

B.C. $\quad X(0) = X(l_x) = 0$

$$Y(0) = Y(l_y) = 0$$

ONLY SIN WILL SATISFY THE B.C.

$$X(x) \sim \sin(px)$$

$$Y(y) \sim \sin(qy)$$

$$p\,l_x = n\pi$$

$$q\,l_y = m\pi$$

$$\boxed{l_x = l_y = l}$$

$$\beta^2 = \frac{\pi^2}{l^2}\left[n^2 + m^2\right]$$

$$\beta = \frac{\pi}{l}\sqrt{n^2 + m^2}$$

$$T(t) \sim \sin\left(\sqrt{n^2+m^2}\,\frac{\pi}{l}\,v\,t\right)$$

$$U_{n,m}(x,y,t) = B_{nm} \sin\left(\sqrt{n^2+m^2}\,\frac{\pi}{\ell}\,v t\right) \sin\left(\frac{\pi}{\ell} n x\right) \sin\left(\frac{\pi}{\ell} m y\right)$$

$$U(x,y,t) = \sum_{n=1} \sum_{m=1} B_{n,m} \sin(\omega_{nm} t) \sin\left(\frac{\pi}{\ell} n x\right) \sin\left(\frac{\pi}{\ell} m y\right)$$

$$\omega_{nm} = \sqrt{n^2+m^2}\,\frac{\pi}{\ell}\,v$$

HARMONICS

U_{11} U_{12} U_{13} U_{23}

U_{21} U_{31} U_{32}

U_{22} U_{33}

TRAVELING WAVE

$$f(x,t) = A\,e^{i\left[\frac{2\pi}{\lambda}(x - vt)\right]}$$

FOR (λ, v). WAVE PROPERTIES

BUT

$$p = \frac{h}{\lambda}$$

a) $$2\pi\frac{x}{\lambda} = \frac{2\pi}{h}\,x p = \frac{x p}{\hbar}$$

$$\hbar \equiv \frac{h}{2\pi}$$

b) $$2\pi\frac{v}{\lambda} = \frac{2\pi}{\lambda}\,\nu\lambda = 2\pi\,\nu$$

$$= \frac{2\pi}{h}\,h\nu = \frac{2\pi}{h}E$$

$$f(x,t) = A\,e^{\frac{i}{\hbar}[px - Et]}$$

FOR (p, E) PARTICLE PROPERTIES

$$\frac{\partial f}{\partial t} = -\frac{i}{\hbar} E f$$

$$-\frac{\hbar}{i}\frac{\partial f}{\partial t} = E f$$

$$\frac{\partial^2 f}{\partial x^2} = \left(\frac{i}{\hbar} p\right)^2 f = -\frac{p^2}{\hbar^2} f$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 f}{\partial x^2} = +\frac{p^2}{2m} f$$

$$= K f = (E - V) f$$

$$\underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2 f}{\partial x^2} + V f}_{\hat{H} f} = E f$$

$$\hat{H} f = -\frac{\hbar}{i}\frac{\partial f}{\partial t}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$$

IF $\hat{H}$ IS TIME INDEPENDENT

$$f(x,t) = e^{-\frac{i}{\hbar}Et}\,\Psi(x)$$

$$-\frac{\hbar}{i}\frac{\partial f}{\partial t} = E\,e^{-\frac{i}{\hbar}Et}\,\Psi(x)$$

$$\hat{H} f = e^{-\frac{i}{\hbar}Et}\,\hat{H}\Psi(x)$$

$$\Rightarrow \boxed{\hat{H}\Psi(x) = E\,\Psi(x)}$$

EIGENVALUE PROBLEM

MATTER $\longleftrightarrow$ WAVE PROPERTIES $\longleftrightarrow$ WAVE EQ.

THE SOLUTIONS OF THIS EQ. $\Longrightarrow$ WAVE FUNCTION

POSTULATE (SCHRÖDINGER)

TIME INDEPENDENT WAVE EQUATION (1dim)

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V(x)\,\psi = E\,\psi$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi = E\,\psi$$

$$\boxed{\hat{H}\psi_{(x)} = E\,\psi_{(x)}}$$

$\hat{H} \equiv$ HAMILTONIAN OPERATOR (ENERGY)

E IS THE EIGENVALUE OF $\hat{H}$

$\psi(x)$ IS THE EIGEN FUNCTION OF $\hat{H}$

CLASSICALLY

$$H = \frac{p^2}{2m} + V(x) = K + V(x)$$

FUNCTION, $f(x)$, IS A SET OF INSTRUCTIONS APPLIED TO THE VARIABLE x.

$$f(x) = 5x^3 + 3x^2 + 2x + 5$$

$\hat{O}$ OPERATOR IS A SET OF INSTRUCTIONS APPLIED TO A FUNCTION

FUNCTION OF FUNCTION

$$\hat{O} f \rightarrow \text{OPERATIONS ON } f$$

OPERATOR

OPERATOR IS A SYMBOL THAT TELLS YOU TO DO SOMETHING TO WHATEVER FOLLOWS THE SYMBOL

EXAMPLE

MULTIPLICATIVE OPERATOR $\quad X$

$$\hat{\Theta} \longrightarrow \frac{d}{dx}, \ \frac{d^2}{dx^2}, \ \int_0^1 dx, \ \sqrt{}$$

$$x\,f(x)$$

$$\hat{\Theta} f(x) \xrightarrow{xf} \frac{df}{dx}, \ \frac{d^2 f}{dx}, \ \int_0^1 f(x)\,dx, \ \sqrt{f(x)}$$

LINEAR OPERATOR

DEF

$$\hat{\Theta}\left(c_1 f(x) + c_2 g(x)\right) \equiv$$

$$c_1 \hat{\Theta} f(x) + c_2 \hat{\Theta} g(x)$$

$\sqrt{}$ IS NOT A LINEAR OPERATOR

SCHRÖDINGER EQ.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi + V(x)\,\Psi = E\,\Psi$$

$$\boxed{\hat{H}\,\Psi = E\,\Psi}$$

EIGENVALUE PROBLEM

GIVEN AN OPERATOR $\hat{O}$

$$\hat{O}\cdot\phi(x) = a\,\phi(x)$$

$a \equiv$ EIGENVALUE

$\phi(x) \equiv$ EIGENFUNCTION

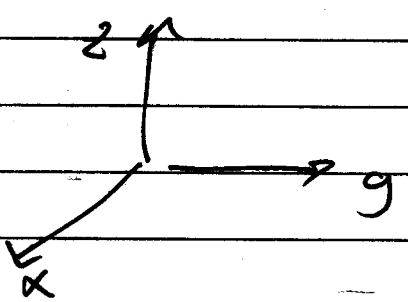
Operators

EIGENFUNCTIONS

ORTHOGONAL FUNCTIONS

COMPLETE SET OF FUNCTIONS

VECTOR SPACE



$$\underset{\sim}{r}_1 = (x_1, y_1, z_1)$$

$$\underset{\sim}{r}_2 = (x_2, y_2, z_2)$$

$$\underset{\sim}{r}_3 = (x_3, y_3, z_3)$$

ORTHOGANALITY

$$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = 0 \Rightarrow \underset{\sim}{r}_1 \perp \underset{\sim}{r}_2$$

$$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 \equiv X_1 X_2 + y_1 y_2 + z_1 z_2$$ Inner Product

$$\langle \underset{\sim}{r}_1 | \underset{\sim}{r}_2 \rangle = \sum_{i=1} X_1 X_2 + y_1 y_2 + z_1 z_2$$