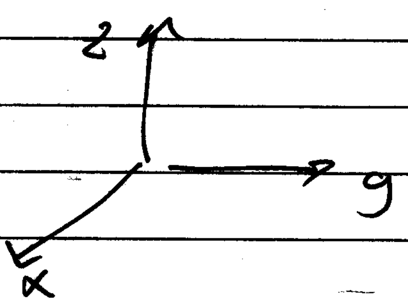
Operators

EIGENFUNCTIONS

ORTHOGONAL FUNCTIONS

COMPLETE SET OF FUNCTIONS

VECTOR SPACE



$$\underset{\sim}{r}_1 = (x_1, y_1, z_1)$$

$$\underset{\sim}{r}_2 = (x_2, y_2, z_2)$$

$$\underset{\sim}{r}_3 = (x_3, y_3, z_3)$$

ORTHOGANALITY

$$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = 0 \Rightarrow \underset{\sim}{r}_1 \perp \underset{\sim}{r}_2$$

$$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 \equiv x_1 x_2 + y_1 y_2 + z_1 z_2$$ Inner Product

$$\langle \underset{\sim}{r}_1 | \underset{\sim}{r}_2 \rangle = \sum_{i=1} x_1 x_2 + y_1 y_2 + z_1 z_2$$

In the space of functions $f_1(x)$ $f_2(x)$

$$\langle f_1 | f_2 \rangle \equiv \int_\Omega f_1^*(x)\, f_2(x)\, dx$$

$\Rightarrow$ $\langle f_1 | f_2 \rangle = 0 \Rightarrow f_1 \perp f_2$

ORTHOGONAL

$\langle f_1 | f_1 \rangle = 1$ Normalized

Consider a set of functions $V = \{f_n\}$

If $\langle f_n | f_m \rangle = \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}$

ORTHONORMAL SET

$$\delta_{mn} \equiv \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}$$

A complet set $\gamma = \{f_n\}$

can expand any other function

$$F(x) = \sum_{n=1}^{\infty} a_n f_n(x)$$

Consider

$$\int_{\Omega} f_j^{*}(x)\, F(x)\, dx$$

$$= \sum_{n=1}^{\infty} a_n \int_{\Omega} f_j^{*}(x)\, f_n(x)\, dx = \sum_{n=1}^{\infty} a_n\, \delta_{jn}$$

$$= a_j$$

$$\boxed{a_j = \int_{\Omega} f_j^{*}(x)\, F(x)}$$