QUANTUM MECHANICS

SIX POSTULATES → 1930

POSTULATE 1

THE STATE OF A QUANTUM MECHANICAL SYSTEM IS COMPLETELY SPECIFIED BY A WAVE FUNCTION $\Psi(x,t)$.

THE PROBABILITY THAT A PARTICLE WILL BE FOUND AT TIME t IN AN SPATIAL INTERVAL OF WIDTH dx CENTERED AT X IS GIVEN BY

$$P(x,t)\,dx = \Psi^*(x,t)\,\Psi(x,t)\,dx$$

$$= |\Psi(x,t)|^2\,dx$$

SINCE THE PARTICLE EXIST A NORMALIZATION CONDITION IS NECESSARY

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2\,dx = 1$$

THEREFORE $\Psi(x,t)$ HAS TO SATISFY SPECIFIC MATHEMATICAL CONSTRAINTS

i) Ψ HAS TO BE SINGLE-VALUED

ii) FIRST DERIVATIVE MUST BE CONTINOUS SO THAT THE SECOND DERIVATIVE EXISTS

iii) SQUARE INTEGRABLE

$\boxed{\text{POSTULATE 2}}$

FOR EVERY OBSERVABLE (MEASORABLE PROPERTY OF THE SYSTEM) IN CLASSICAL MECHANICS (POSITION, MOMENTUM, AND ENERGY) THERE EXISTS A CORRESPONDING OPERATOR IN Q.M.

OPERATORS $\longleftrightarrow$ HERMITIAN $\longleftrightarrow$ $\lambda \in \mathbb{R}$

$$p \longleftrightarrow -i\hbar \frac{\partial}{\partial x}$$

$$x \longleftrightarrow x$$

$$E \longleftrightarrow -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

$\Rightarrow$ AFTER Post.3

POSTULATE 3

IN ANY SINGLE MEASUREMENT OF THE OBSERVABLE THAT CORRESPONDS TO THE OPERATOR $\hat{A}$, THE ONLY VALUES THAT WILL BE EVER MEASURED ARE THE EIGENVALUES OF THE OPERATOR

$$\hat{H}\Psi_n = E_n \Psi_n$$

$E_n \in \mathbb{R}$

THE WAVE FUNCTION DOES NOT NEED TO BE AN EIGENFUNCTION OF AN OPERATOR $\hat{A}$.

POSTULATE 4

THE VALUE OF THE OBSERVABLE a IS MEASURED ONCE EACH ON MANY IDENTICAL PREPARED SYSTEMS, THE AVERAGE VALUE (EXPECTATION VALUE) OF ALL OF THESE MEASUREMENTS IS GIVEN BY

$$\langle A \rangle = \frac{\int_{-\infty}^{\infty} \Psi^*(x,t)\, \hat{A}\, \Psi(x,t)\, dx}{\int_{-\infty}^{\infty} \Psi^*(x,t)\, \Psi(x,t)\, dx}$$

IN THE CASE $\Psi(x,t) = \phi_j(x,t)$

$$\hat{A}\,\phi_j(x,t) = a_j\,\phi_j(x,t)$$

$$\langle A \rangle = a_j$$

IN THE CASE $\Psi(x,t) \neq \phi_j(x,t) \quad \forall j$

$$\Psi(x,t) = \sum_{n=1}^{\infty} b_n\,\phi_n(x,t)$$

SUCH THAT

$$\int_{-\infty}^{\infty} \Psi^*(x,t)\,\Psi(x,t)\,dx = 1$$

THEREFORE

$$\langle A \rangle = \int_{-\infty}^{\infty} \sum_m b_m^*\,\phi_m^*(x,t) \sum_n b_n\,\phi_n(x,t)\,dx$$

$$= \sum_m \sum_n b_m^*\,b_n \int_{-\infty}^{\infty} \phi_m^*(x,t)\,\phi_n(x,t)\,dx$$

$$\int_{-\infty}^{\infty} \phi_m^*(x,t)\, \phi_n(x,t)\, dx = \delta_{mn}$$ ORTHOGONAL

$$\langle A \rangle = \sum_m |b_m|^2 a_n$$ AVERAGE

weighted average ⟷ Measure the frequencies of λ_i

WE CAN NOT KNOW THE OUTCOME OF AN INDIVIDUAL MEASUREMENT
BUT THE ACT OF CARRYING OUT A QM MEASUREMENT APPEARS TO CONVERT THE WAVE FUNCTION OF A SYSTEM TO AN EIGENFUNCTION OF THE OPERATOR CORRESPONDING TO THE MEASURED QUANTITY

POSTULATE ⟷ THE MEASUREMENT HAS TO BE CARRIED OUT ONLY ONCE ON A LARGE NUMBER OF IDENTICALLY PREPARED SYSTEMS.

GIVEN A WAVE FUNCTION $\Psi(x,t)$ AND AN OPERATOR $\hat{A}$, WE WANT TO KNOW THE AVERAGE VALUE OF THE OBSERVABLE ASSOCIATED WITH $\hat{A}$.

$$\langle A \rangle = \frac{\int_V \Psi^*_{(x,t)} \left(\hat{A}\Psi\right) dV}{\int_V \Psi^* \Psi \, dV}$$

WE CAN DO BETTER IF WE FIRST SOLVE THE FOLLOWING EIGENVALUE PROBLEM

$$\hat{A}\phi_n = \lambda_n \phi_n$$

SINCE $\hat{A}$ IS HERMITIAN $(\lambda_n \in \mathbb{R})$ THE SET $\{\phi_n\}$ IS COMPLETE THUS

$$\Psi = \sum b_n \phi_n$$

AND

$$\int_V \phi_m^* \phi_n \, dV = \delta_{m,n}$$

FOR SIMPLICITY ASSUME THAT

$$\int_V \Psi^* \Psi \, dV = 1$$

$$\langle \hat{A} \rangle = \int_V \Psi^* (\hat{A}\Psi) dV$$

$$\boxed{\langle \hat{A} \rangle = \sum_m |b_m|^2 \lambda_n}$$

SINCE Ψ IS NORMALIZED

$$\sum_m |b_m|^2 = 1$$