POSTULATE 5

THE EVOLUTION IN TIME OF A QM SYSTEM IS GOVERNED BY A TIME-DEPENDENT SCHRÖDINGER EQUATION:

$$\boxed{\hat{H}\,\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial \tau}}$$

IF $\hat{H}$ IS TIME INDEPENDET (NO EXPLICIT TIME DEPENDANCE)

$$\Psi(x,t) = e^{\frac{iE}{\hbar}t}\,\phi(x)$$

$$\boxed{\hat{H}\,\phi(x) = E\,\phi(x)}$$

PREPARE MULTIPLE COPIES OF A ONE HYDROGEN SYSTEM AND MEASURE THE ENERGY

$$\Psi = \sum b_n \phi_n$$

WHERE $\hat{H}\phi_n = E_n \phi_n$

BEFORE THE MEASUREMENT WE HAVE

$$\Psi = \sum_n b_n \phi_n$$

AFTER THE MESUREMENT (E_n) WE HAVE

$$\phi_n.$$

WE HAVE COLLAPSED THE WAVE FUNCTION.

THE $|b_m|^2$S ARE THE PROBABILITIES OF OBSERVING λ_n. IN THE LAB THE $|b_m|^2$S CAN BE CALCULATED FROM THE MEASUREMENTS OF OBSERVABLE A.

CONSIDER m

$$\boxed{\int_V \phi_m^* \Psi \, dV = \sum_l \left[\int_V \phi_m^* \phi_l \, dV\right] b_l = b_m}$$

THUS

$$\boxed{\Psi = \sum_m \phi_m \int_V \phi_m^* \Psi \, dV}$$

USING THE KET NOTATION

$$\phi_m \equiv |m\rangle \qquad \Psi \equiv |\Psi\rangle$$

$$\int_V \phi_m^* \Psi \, dV \equiv \langle m | \Psi \rangle$$

$$|\Psi\rangle = \sum_m |m\rangle\langle m|\Psi\rangle$$

NORMALIZATION

$$\Rightarrow \langle\Psi|\Psi\rangle = 1$$

$$\langle\hat{A}\rangle = \langle\Psi|A\Psi\rangle$$

$$\langle\Psi| = \sum_l \langle\Psi|m\rangle\langle m|$$

$$\langle\hat{A}\rangle = \sum_{lm} \langle\Psi|l\rangle\underbrace{\langle l|Am\rangle}_{\substack{\lambda_m\langle l|m\rangle \\ \lambda_m\delta_{ml}}}\langle m|\Psi\rangle$$

$$= \sum_m \langle\Psi|m\rangle\lambda_m\langle m|\Psi\rangle$$

$$\boxed{\langle A \rangle = \sum_m |\langle m|\Psi\rangle|^2 \lambda_m}$$

<u>SINCE</u> $\langle \Psi | m \rangle = \langle m | \Psi \rangle^*$

$$|\Psi\rangle = \sum_m |m\rangle\langle m|\Psi\rangle$$

$$\hat{\mathbb{I}} \equiv \sum_m |m\rangle\langle m|$$

$$|\Psi\rangle = \hat{\mathbb{I}}|\Psi\rangle$$

$\boxed{\text{FREE PARTICLE}}$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

FOR A FREE PARTICLE $V(x) = 0$

$$\hat{H}\Psi = E\Psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} = E\Psi$$

$$\Rightarrow \quad \frac{d^2\Psi}{dx^2} = -\frac{2mE}{\hbar^2}\Psi$$

$$k^2 \equiv \frac{2mE}{\hbar^2}$$

$$\Psi^+(x) = A^+ e^{ikx}$$

$$\Psi^-(x) = A^- e^{-ikx}$$

$$k = \frac{2\pi}{\lambda} = \sqrt{\frac{2mE}{\hbar^2}}$$

$$k = \frac{\sqrt{2mE}}{\hbar} = \frac{2\pi\sqrt{2mE}}{h}$$

$$= 2\pi \frac{p}{h}$$

$$\boxed{kh = 2\pi p} = 2\pi \frac{h}{\lambda}$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

$$\boxed{\hbar k = p}$$

$$|\Psi^+|^2 = |A^+|^2 = \text{CONSTANT}$$

IN 1-D
$$\int_{-\infty}^{\infty} |\Psi^+|^2 dx = \rightarrow \infty!$$

PICK A VERY LARGE BOX

$$\int_{-L}^{L} |\Psi^+|^2 dx = 2L\,|A^+|^2$$

PICK $\boxed{A^+ = \frac{1}{\sqrt{2L}}}$

$$P(x)dx = \frac{|A^+|^2 dx}{\int_{-L}^{L} |\Psi^+|^2 dx} = \frac{dx}{2L}$$

$$\xrightarrow[L \to \infty]{} 0$$

WE REALLY DO NOT KNOW WHERE IS THE PARTICLE BUT WE KNOW THE MOMENTUM

$$p = \hbar k$$