NORMALIZED WAVE FUNCTION

$$\boxed{\Psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(n\frac{\pi}{L}x\right)}$$

SINCE $P(x)\,dx$ IS THE PROBABILITY OF FINDING THE PARTICLE BETWEEN $x$ AND $x+dx$, WHAT IS THE AVERAGE POSITION, $\bar{x}$.

$$\bar{x} = \int_0^L x\,P(x)\,dx \equiv \langle x\rangle$$ EXPECTED VALUE

$$\langle x\rangle = \frac{2}{L}\int_0^L x\sin^2\left(\frac{n\pi}{L}x\right)dx = \frac{L}{2}$$

$$\boxed{\langle x\rangle = \frac{L}{2}}$$

$$\langle x^2\rangle = \frac{2}{L}\int_0^L x^2\sin^2\left(\frac{n\pi}{L}x\right)dx =$$

$$\boxed{\langle x^2\rangle = \frac{L^2}{3} - \frac{L^2}{2\pi^2}\frac{1}{n^2}}$$

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\boxed{\sigma_x^2 = \left(\frac{L}{2\pi n}\right)^2 \left[\frac{\pi^2 n^2}{3} - 2\right]}$$

HOW ABOUT MOMENTUM?

MOMENTUM IS AN OPERATOR

$$\hat{H} \Psi = E \Psi$$

$$\Psi^* (\hat{H} \Psi) = E \, \Psi^* \Psi$$

$$\int \Psi^* (\hat{H} \Psi) \, dx = E \underbrace{\int \Psi^* \Psi \, dx}_{1}$$

$$\int \Psi^* (\hat{H} \Psi) \, dx = E$$

THUS FOR AN OPERATOR $\hat{O}$, THE EXPECTED VALUE IS GIVEN BY

$$\boxed{\langle \hat{O} \rangle \equiv \int \Psi^* (\hat{O} \Psi) \, dx}$$

NOW CONSIDER $\hat{P}_x = -i\hbar \frac{d}{dx}$

$$\langle p \rangle = \int_0^L \Psi_n^*(x) \left( -i\hbar \frac{d}{dx} \Psi(x) \right) dx$$

$$= \frac{2}{L} \left( -i\hbar \right) n \frac{\pi}{L} \int_0^L \sin\left(n\frac{\pi}{L}x\right) \cos\left(n\frac{\pi}{L}x\right) dx$$

$$\boxed{\langle p \rangle = 0}$$

$$\langle p^2 \rangle = \int_0^L \Psi_n^*(x) \left( \hat{P}_x \left( \hat{P}_x \Psi(x) \right) \right) dx$$

$$= -\hbar^2 \int_0^L \Psi_n^*(x) \left( \frac{d^2}{dx^2} \Psi(x) \right) dx$$

$$= +\hbar^2 \left( \frac{n\pi}{L} \right)^2 \frac{2}{L} \int_0^L \sin\left(n\frac{\pi}{L}x\right) \sin\left(n\frac{\pi}{L}x\right) dx$$

$$\boxed{\langle p^2 \rangle = \frac{\hbar^2 \pi^2}{L^2} n^2}$$

$$\boxed{\sigma_p^2 = \langle p^2 \rangle - \langle p \rangle^2 = \frac{\hbar^2 \pi^2}{L^2} n^2}$$

$$\sigma_p = \frac{\hbar \pi}{L} n$$

$$\sigma_x = \frac{L}{2\pi n} \sqrt{\frac{\pi^2 n^2}{3} - 2}$$

$$\boxed{\sigma_x \sigma_p = \frac{\hbar}{2} \sqrt{\frac{\pi^2 n^2}{3} - 2} > \frac{\hbar}{2}}$$

UNCERTAINTY PRINCIPLE

$$\langle \hat{H} \rangle = \left\langle \frac{\hat{p}^2}{2m} \right\rangle = \frac{1}{2m} \langle p^2 \rangle$$

$$= E_n = \frac{1}{2m} \frac{\hbar^2 \pi^2}{L^2} n^2$$

$$E_n = \frac{h^2}{8mL^2} n^2$$

# Particle in the Box

Normalization condition

**Integrate** $\left[A^2 \text{ Sin}\left[\frac{m\pi x}{L}\right]^2, \{x, 0, L\}, \text{Assumptions} \to \{m \in \text{Integers}\}\right]$

$\frac{A^2 L}{2}$

Normalized eigen function

$\Psi[\text{x\_}] := \sqrt{\frac{2}{L}} \text{Sin}\left[\frac{m\pi x}{L}\right]$

Average position < x >

**xbar = Integrate** $\left[x\Psi[x]^2, \{x, 0, L\}, \text{Assumptions} \to \{m \in \text{Integers}\}\right]$

$\frac{L}{2}$

Average of the square position $< x^2 >$

**x2bar = Integrate** $\left[x^2\Psi[x]^2, \{x, 0, L\}, \text{Assumptions} \to \{m \in \text{Integers}\}\right]$

$\frac{1}{6}L^2\left(2 - \frac{3}{m^2\pi^2}\right)$

Average momentum

**po[f_]:=** $-ih\partial_x f$

**pbar = Integrate** $\left[\text{Sin}\left[\frac{m\pi x}{L}\right] \text{po}[\Psi[x]], \{x, 0, L\},\right.$
$\left.\text{Assumptions} \to \{m \in \text{Integers}\}\right]$

0

Average of the square of the momentum $< p^2 >$

**p2bar = Integrate**$[\Psi[x]\text{po}[\text{po}[\Psi[x]]], \{x, 0, L\}, \text{Assumptions} \to \{m \in \text{Integers}\}]$

$\frac{h^2 m^2 \pi^2}{L^2}$

$\Delta\text{x2} = \text{x2bar} - \text{xbar}^2$

$-\frac{L^2}{4} + \frac{1}{6}L^2\left(2 - \frac{3}{m^2\pi^2}\right)$

$\Delta\text{p2} = \text{p2bar} - \text{pbar}^2$

$\frac{h^2 m^2 \pi^2}{L^2}$

$\Delta\text{p}\Delta\text{x} = \text{Simplify}[\Delta\text{p2 } \Delta\text{x2}]//\text{Sqrt}$

$\frac{\sqrt{h^2(-6+m^2\pi^2)}}{2\sqrt{3}}$

$(\Delta p\ \Delta x//\text{Simplify})/.\{h \to 1, m \to 1\}//N$

0.567862

$1/(8.\pi)$

0.0397887