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PARTICLE IN A FINITE BOX

$$V(x) = \begin{cases} V_0 & x \leq 0 & \text{(I)} \\ 0 & 0 < x < L & \text{(II)} \\ V_0 & L \leq x & \text{(III)} \end{cases}$$

WE DIVIDE THE PHYSICAL SPACE IN 3 REGIONS: I, II, III.

FROM THE TISE AND $E < V_0$

$$-\frac{\hbar^2}{2m} \frac{d^2 \Psi}{dx^2} + V(x) \Psi = E \Psi$$

WE GET

$$\frac{d^2 \Psi}{dx^2} = + \frac{2m(V_0 - E)}{\hbar^2} \Psi$$

IN REGIONS I AND III

$$\frac{d^2 \Psi}{dx^2} = k^2 \Psi,$$

WHERE

$$k^2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

BUT IN REGION II

$$\frac{d^2 \Psi}{dx^2} = -k^2 \Psi,$$

WITH

$$k^2 = \sqrt{\frac{2mE}{\hbar^2}}$$

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SOLUTIONS IN REGION I, II, AND III

$$\Psi_I(x) = A e^{kx} + B e^{-kx}$$

$$\Psi_{II}(x) = C e^{ikx} + D e^{-ikx}$$

$$\Psi_{III}(x) = E e^{kx} + F e^{-kx}$$

BUT AT $x \rightarrow \infty$ AND $x \rightarrow -\infty$, WE SHOULD NOT FIND THE PARTICLE, THUS

$$B = 0 = F.$$

NEXT, WE CONSIDER CONTINUITY OF THE FUNCTIONS Ψ AND ITS FIRST DERIVATIVE

$$\Psi_I(0) = \Psi_{II}(0)$$

$$\Psi_{II}(L) = \Psi_{III}(L)$$

$$\left. \frac{d\Psi_I}{dx} \right|_{x=0} = \left. \frac{d\Psi_{II}}{dx} \right|_{x=0}$$

$$\left. \frac{d\Psi_{II}}{dx} \right|_{x=L} = \left. \frac{d\Psi_{III}}{dx} \right|_{x=L}$$

Table 6-2. A Summary of the Systems Studied in Chapter 6

Name of System	Physical Example	Potential and Total Energies	Probability Density	Significant Feature
Zero potential	Proton in beam from cyclotron			Results used for other systems
Step potential (energy below top)	Conduction electron near surface of metal			Penetration of excluded region
Step potential (energy above top)	Neutron trying to escape nucleus			Partial reflection at potential discontinuity
Barrier potential (energy below top)	alpha particle trying to escape Coloumb barrier			Tunneling
Barrier potential (energy above top)	Electron scattering from negatively ionized atom			No reflection at certain energies
Finite square well potential	Neutron bound in nucleus			Energy quantization
Infinite square well potential	Molecule strictly confined to box			Approximation to finite square well
Simple harmonic oscillator potential	Atom of vibrating diatomic molecule			Zero-point energy

QUESTIONS

- Can there be solutions with $E < 0$ to the time-independent Schroedinger equation for the zero potential?
- Why is it never possible in classical mechanics to have $E < V(x)$? Why is it possible in quantum mechanics, providing there is some region in which $E > V(x)$?
- Explain why the general solution to a one-dimensional time-independent Schroedinger equation contains two different functions, while the general solution to the corresponding Schroedinger equation contains many different functions.
- Consider a particle in a long beam of very accurately known momentum. Does a wave function in the form of a group provide a more or a less realistic description of the particle than a single complex exponential wavefunction like (6-9)?

- Under what conditions is reflection of a wave function due to a discontinuity in the potential?
- If a potential has a discontinuity, what is the relationship between the wave function and its derivative at the discontinuity?
- By combining the wave function and its derivative at a discontinuity, what is the relationship between the wave function and its derivative at the discontinuity?
- Just what is a wave function?
- How can it be shown that the wave function is associated with a probability?
- Is there an apparatus for measuring the position of an unbound particle? Can it be followed by an apparatus? Why or why not?
- Exactly what is the relationship between the wave function and the probability of finding a particle in a certain region? How is this relationship established?
- Since a real wave function is oscillatory, how is it possible to have a probability density that is not oscillatory?
- What do you mean by a wave packet? How is it related to the wave function?
- What is the relationship between the wave function and the probability of finding a particle in a certain region? How is this relationship established?
- A particle is incident on a potential barrier. What is the probability of reflection? What is the probability of transmission?
- In the sun, the temperature is about 10^7 K. The Coulomb barrier between two protons is about 1 MeV. How is it possible for the sun to shine? What is the mechanism?
- Are there any particles which are strictly confined to a region? If so, what are they?
- Show from the uncertainty principle that a particle always has a non-zero energy. What is the minimum energy?
- Why do finite square wells have discrete energy levels? What is the relationship between the energy levels and the width of the well?
- What would the wave function look like for a particle in a finite square well? How does it change as the energy increases?
- Why do the energy levels of a finite square well approach those of an infinite square well as the width of the well increases? What is the relationship between the energy levels and the width of the well?
- In the $n = 3$ state, how many nodes does the wave function have? How does the number of nodes change as n increases?
- Explain in simple terms the relationship between the wave function and the probability of finding a particle in a certain region. How is this relationship established?
- Would you expect the wave function of a particle in a potential well to be oscillatory? Why or why not?
- If the wave function of a particle in a potential well is oscillatory, what is the relationship between the energy and the frequency of the oscillation? How is this relationship established?