PARTICLE IN A FINITE BOX

$$
V(x) = \begin{cases} V_0 & x \leq 0 \quad (\mathrm{I}) \\ 0 & 0 < x < L \quad (\mathrm{II}) \\ V_0 & L \leq x \quad (\mathrm{III}) \end{cases}
$$

WE DIVIDE THE PHYSICAL SPACE IN 3 REGIONS: I, II, III.

FROM THE TISE AND $E < V_0$

$$
-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V(x)\Psi = E\Psi
$$

WE GET

$$
\frac{d^2\Psi}{dx^2} = +\frac{2m}{\hbar^2}(V_0 - E)\Psi
$$

IN REGIONS I AND III

$$
\frac{d^2\Psi}{dx^2} = \kappa^2\Psi,
$$

WHERE

$$
\kappa^{\cancel{2}} = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}
$$

BUT IN REGION II

$$
\frac{d^2\Psi}{dx^2} = -k^2\Psi,
$$

WITH

$$
k^{\cancel{2}} = \sqrt{\frac{2mE}{\hbar^2}}
$$

SOLUTIONS IN REGION I, II, AND III

$$\Psi_{I}(x) = A e^{kx} + \underline{B} e^{-kx}$$

$$\Psi_{II}(x) = C e^{ikx} + D e^{-ikx}$$

$$\Psi_{III}(x) = \underline{E} e^{kx} + F e^{-kx}$$

BUT AT $x \to \infty$ AND $x \to -\infty$, WE SHOULD NOT FIND THE PARTICLE, THUS

$$B = 0 = E.$$

NEXT, WE CONSIDER CONTINUITY OF THE FUNCTIONS Ψ AND ITS FIRST DERIVATIVE

$$\Psi_{I}(0) = \Psi_{II}(0)$$

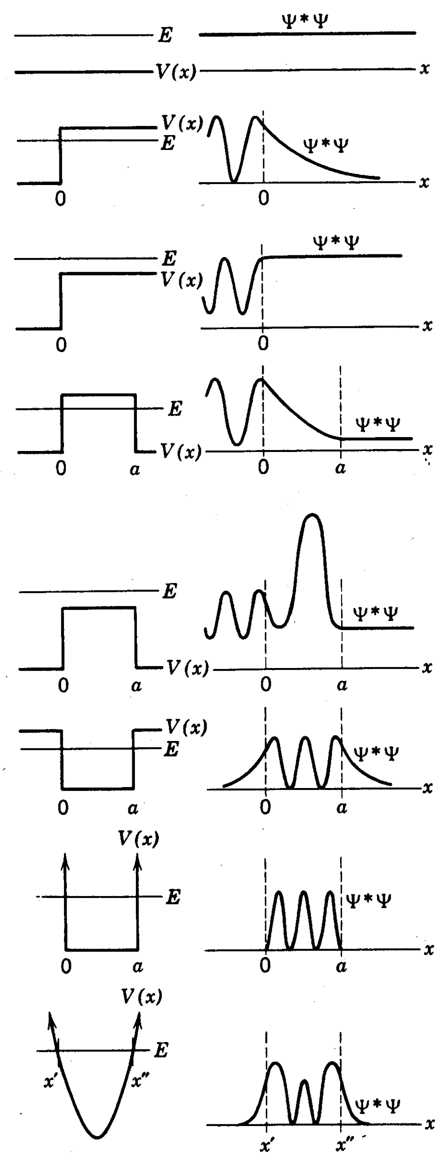
$$\Psi_{II}(L) = \Psi_{III}(L)$$

$$\left.\frac{d\Psi_{I}}{dx}\right|_{x=0} = \left.\frac{d\Psi_{II}}{dx}\right|_{x=0}$$

$$\left.\frac{d\Psi_{II}}{dx}\right|_{x=L} = \left.\frac{d\Psi_{III}}{dx}\right|_{x=L}$$

Table 6-2. A Summary of the Systems Studied in Chapter 6

Name of System	Physical Example	Potential and Total Energies	Probability Density	Significant Feature
Zero potential	Proton in beam from cyclotron			Results used for other systems
Step potential (energy below top)	Conduction electron near surface of metal			Penetration of excluded region
Step potential (energy above top)	Neutron trying to escape nucleus			Partial reflection at potential discontinuity
Barrier potential (energy below top)	α particle trying to escape Coloumb barrier			Tunneling
Barrier potential (energy above top)	Electron scattering from negatively ionized atom			No reflection at certain energies
Finite square well potential	Neutron bound in nucleus			Energy quantization
Infinite square well potential	Molecule strictly confined to box			Approximation to finite square well
Simple harmonic oscillator potential	Atom of vibrating diatomic molecule			Zero-point energy



QUESTIONS

1. Can there be solutions with $E < 0$ to the time-independent Schroedinger equation for the zero potential?
2. Why is it never possible in classical mechanics to have $E < V(x)$? Why is it possible in quantum mechanics, providing there is some region in which $E > V(x)$?
3. Explain why the general solution to a one-dimensional time-independent Schroedinger equation contains two different functions, while the general solution to the corresponding Schroedinger equation contains many different functions.
4. Consider a particle in a long beam of very accurately known momentum. Does a wave function in the form of a group provide a more or a less realistic description of the particle than a single complex exponential wavefunction like (6-9)?

5. Under what c mation to an
6. If a potential discontinuities
7. By combining ing wave. Wh
8. Just what is a
9. How can it b associated pa
10. Is there an an ior of an unb can be follow apparatus? W
11. Exactly what incident on a by the statem than the step
12. Since a real e oscillatory fu
13. What do you 6-8 as it refle
14. What is the fa tunneling thr
15. A particle is i and it is refle direction of i changed into
16. In the sun, tv the Coulomb than the sum is responsible on earth if it
17. Are there any which is of ir
18. Show from a always has o the eigenfunc
19. Why do finit the character
20. What would well look lik
21. Why do the best approxi well?
22. In the $n = 3$ positions bet positions?
23. Explain in si principle.
24. Would you matter at ve
25. If the eigenfu has even par