16.1, 2, 3, 5, 7

$$\hat{p}_x \quad \hat{x}$$

$$\hat{x}\,\hat{p}_x \qquad\qquad \hat{p}_x = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{p}_x\,\hat{x}$$

$$\hat{x}\hat{p}_x f = -i\hbar\, x\frac{\partial f}{\partial x}$$

$$\hat{p}_x\hat{x} f = -i\hbar\frac{\partial}{\partial x}(xf) = -i\hbar x\frac{\partial f}{\partial x} - i\hbar f$$

$$(\hat{x}\hat{p}_x - \hat{p}_x\hat{x})f = i\hbar f$$

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{C}$$

$$\boxed{[\hat{x}, \hat{p}_x] = i\hbar\hat{I}}$$

$$[\hat{x}, \hat{p}_y] = 0$$

$$[\hat{A}, \hat{B}^2] = \hat{A}\hat{B}^2 - \hat{B}^2\hat{A} - \hat{B}\hat{A}\hat{B} + \hat{B}\hat{A}\hat{B}$$

$$\boxed{[\hat{A}, \hat{B}^2] = [\hat{A}, \hat{B}]\hat{B} + \hat{B}[\hat{A}, \hat{B}]}$$

OPERATORS

$$\hat{A} f_a = \lambda_a f_a$$

$$\hat{B} g_b = \lambda_b g_b$$

IS f_a AN EIGENFUNCTION OF $\hat{B}$

$$\hat{B} f_a = \lambda f_a \; ?$$

IF

$$\hat{B} f_a = \beta_a f_a$$

THEN $\hat{A}$ AND $\hat{B}$ COMMUTE

$$\hat{A} \hat{B} f_a = \hat{A} (\hat{B} f_a) = \beta_a (\hat{A} f_a)$$

$$= \beta_a \lambda_a f_a$$

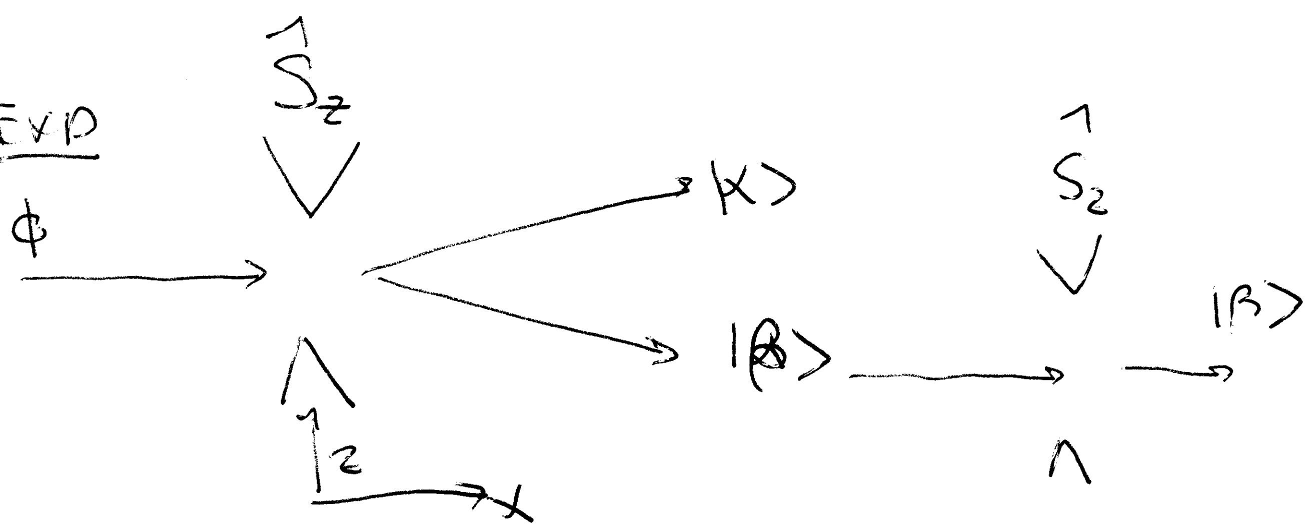
$$\hat{B} \hat{A} f_a = \beta_a \lambda_a f_a$$

AND $$\left(\hat{A}\hat{B} - \hat{B}\hat{A}\right) f_n = 0$$

<u>DEF</u> $$[\hat{A}, B] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}$$

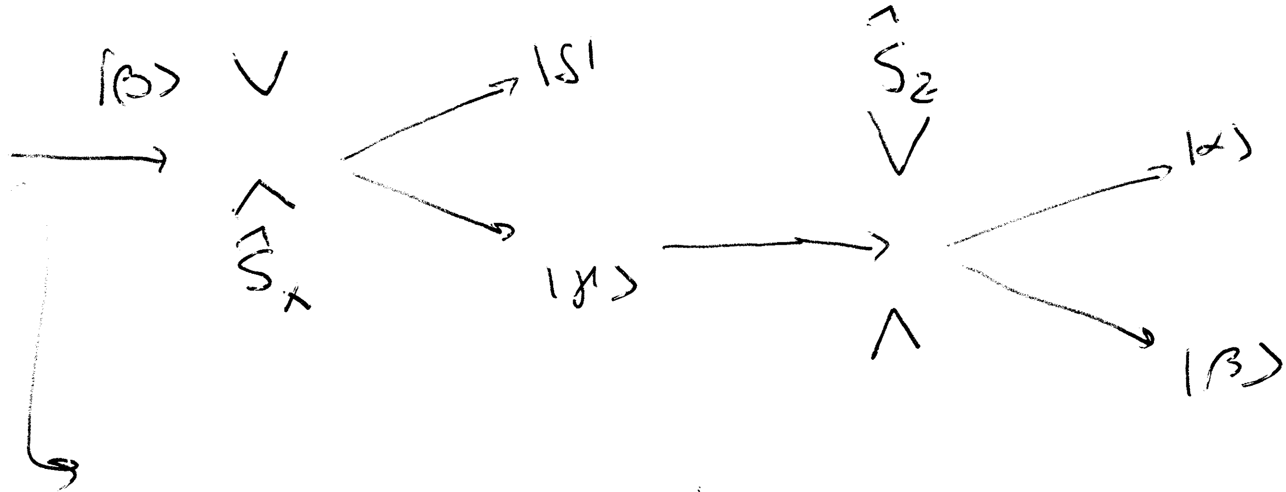
<u>THEOREM</u> $\hat{A}$ AND $\hat{B}$ COMMUTE IF AND ONLY IF THEY HAVE COMMON EIGEN FUNCTIONS.

<u>EXP</u>



$$\phi = c_1 |\alpha\rangle + c_2 |\beta\rangle$$

$$|c_1|^2 + |c_2|^2 = 1$$



$$|\beta\rangle = d_1|\delta\rangle + d_2|\gamma\rangle$$

$$|d_1|^2 + |d_2|^2 = 1$$

$|\alpha\rangle, |\beta\rangle$ EIGEN FUNCTIONS $\hat{S}_z$

BUT THEY ARE NOT EIGEN FUNCTIONS $\hat{S}_x$

$$[\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}$$

IF $[\hat{A}, \hat{B}] = 0 \Longleftrightarrow \hat{A}$ AND $\hat{B}$ HAVE COMMON EIGENFUNCTIONS.

IN GENERAL

$$\sigma_A^2 \, \sigma_B^2 > \frac{1}{4} \left| \int \psi^*(x) [\hat{A}, \hat{B}] \psi(x) \, dx \right|^2$$

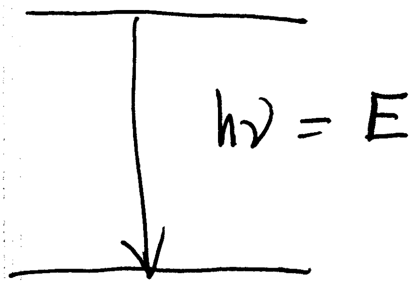
$$\sigma_A \equiv \langle (\hat{A} - \langle \hat{A} \rangle)^2 \rangle$$

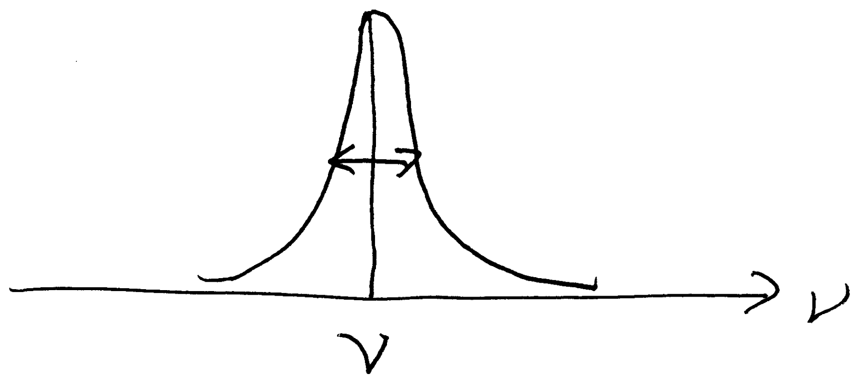
$$\sigma_B \equiv \langle (\hat{B} - \langle \hat{B} \rangle)^2 \rangle$$

$$\boxed{\sigma_A \, \sigma_B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle \right|}$$

$$\Delta p_x \, \Delta x \geq \frac{1}{2} \hbar$$

$$\boxed{\Delta E \, \Delta t \geq \frac{1}{2} \hbar \quad ?}$$





$\Delta E \, \Delta \tau \sim \hbar$

$\Delta t \sim \dfrac{\hbar}{\Delta E}$ HALFLIFE OF EXCITED STATE

LOOK AT FREE PARTICLE

$$\psi(x) = A e^{ikx}$$ FOR A BOX SIZE 2L

$$\hat{p}\psi = \hbar k \psi$$ EXACT MOMENTUM

$$\Delta p = 0$$

$$\Rightarrow \Delta x \to \infty$$

$$P(x) = \frac{1}{2L}$$ SAME FOR ANY X.

LET US ASSUME $\Delta p = \hbar \Delta k \neq 0$

$$k = k_0 + n\Delta k$$ WITH $\Delta k << k_0$

$$\Psi(x) = \frac{1}{2} A e^{ik_0 x} + \sum_{n=-N}^{N} \frac{1}{2} A e^{i(k_0 + n\Delta k)x}$$

$$k \in \langle k_0 - N\Delta k,\ k_0 + N\Delta k \rangle$$

$$L = 3.14 \times 10^{-10}\ m$$

$$k_0 = 7.00 \times 10^{10}\ m^{-1}$$

$$N = 10$$