$$\Psi(x) = \frac{1}{\sqrt{2L}} e^{ikx} \Rightarrow \begin{array}{l} \Delta p = 0 \\ \Delta x = \infty \end{array}$$

---

$$k = k_0 + n\Delta k \qquad \Delta k \ll k_0$$

$$\Psi(x) = \frac{1}{2\sqrt{2L}} e^{ik_0 x} + \sum_{n=-N}^{N} \frac{1}{2} \frac{1}{\sqrt{2L}} e^{i(k_0 + n\Delta k)x}$$

$$\Delta p = \hbar \Delta k$$

$$\Delta x < \infty$$

---

$$\Psi(x) = \frac{1}{2} \frac{1}{\sqrt{2L}} e^{ik_0 x} + \sum_{n=-\infty}^{\infty} \frac{1}{2} \frac{1}{\sqrt{2L}} e^{i(k_0 + n\Delta k)x}$$

$$\Rightarrow \quad \Psi(x) = \frac{1}{\sqrt{2L}} \sum_{k=-\infty}^{\infty} e^{ikx} \qquad \begin{array}{l} \Delta p = \infty \\ \Rightarrow \Delta x = 0 \end{array}$$

---

WHAT DO WE GET WHEN $N \to \infty$

$$\sum_{k=-\infty}^{\infty} e^{ikx} = ?$$

TOTAL LOCALIZATION
$\Delta x = 0$

$\Delta p = \infty$

$$\sum_{k=-\infty}^{\infty} e^{ik(x-x_0)} \sim \delta(x-x_0)$$

DIRAC'S DELTA

$$\delta(x-x_0) = \begin{cases} 0 & x \neq x_0 \\ \infty & x = x_0 \text{ BUT } \int_{-\infty}^{\infty} \delta(x-x_0)\,dx = \end{cases}$$

$$\hat{O} f_n = \lambda_n f_n$$

$$\{f_n\}$$

$$\Psi(x) = \sum_n A_n f_n$$

$$\langle f_n | \Psi \rangle = \int_\Omega f_n^*(x') \Psi(x')\, dx' = A_n$$

$$\Psi(x) = \sum_n \int_\Omega f_n^*(x') \Psi(x')\, dx' \, f_n$$

$$= \int_\Omega \underbrace{\sum_n f_n^*(x') f_n(x)}_{\delta(x-x')} \Psi(x')\, dx'$$

$\boxed{\text{PIB}}$

MOMENTUM REPRESENTATION

$$\Psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right) & 0 < x < L \\ 0 & x < 0 \quad \cancel{|x| > L} \\ & x > L \end{cases}$$

$$\Psi_n(x) = \sum_{k=-\infty}^{\infty} A_k^n e^{ikx}$$

$$\hat{p}\Psi_n(x) = \sum_{k=-\infty}^{\infty} \hbar k A_k^n e^{ikx}$$

$$\langle \Psi_n | \hat{p} \Psi_n \rangle = \sum_{k=-\infty}^{\infty} |A_k^n|^2 \hbar k$$

$$= \int_0^L \Psi_n^*(x)\, \hat{p}\, \Psi_n(x)\, dx = 0$$

$$|A_k^n|^2 = |A_{-k}^n|^2$$

$$\int_{-\infty}^{\infty} e^{-ikx'} \Psi_n(x)\, dx = \sum_{k=-\infty}^{\infty} A_k^n \underbrace{\int_{-\infty}^{\infty} e^{-ikx'} e^{ikx}\, dx}_{\cancel{\delta(x-x')}\ \leftarrow\ \delta_{kk'}}$$

$$p = \hbar k$$

$$E = \frac{h^2}{8mL^2} n^2$$

$$p = \sqrt{2mE}$$

$$p = \sqrt{\frac{h^2}{4L^2}}\, n$$

$$p = \frac{h}{4L} n = \hbar \frac{\pi}{L} n = \hbar k_n$$

$$k_n = \frac{\pi}{L} n \rightarrow \frac{\pi}{L} \quad \frac{\pi}{L}5 \quad \frac{\pi}{L}15 \quad \frac{\pi}{L}101$$

$$\int_{-\infty}^{\infty} e^{-ik'x}\,\Psi_n(x)\,dx = A^n_{k'} \equiv \langle k' | \Psi_n \rangle$$

$$A^n_k \equiv \int_{-\infty}^{\infty} e^{-ikx}\,\Psi_n(x)\,dx = \int_0^L e^{-ikx}\sqrt{\tfrac{2}{L}}\,\sin\left(\tfrac{n\pi}{L}x\right)dx$$

$$\left.\begin{aligned} A^n_{-k} &= A^{n*}_k \\ A^n_k &= A^{n*}_{-k} \end{aligned}\right\} \quad |A^n_k|^2 = A^n_k A^{n*}_k = A^{n*}_{-k} A^n_{-k} = |A^n_{-k}|^2$$

$A^n_k$ = is the MOMENTUM REPRESENTATION OF $\Psi_n(x) = \sqrt{\tfrac{2}{L}}\sin\left(\tfrac{n\pi}{L}x\right)$

$|A^n_k|^2$ probability density of finding the particle with momentum $\hbar k$

For a 50 nm box

```
In[1066]:= phiS[n_][x_] := Sqrt[2/50] Sin[n π x/50]

In[1067]:= fp[x_, k_] := Exp[ ⅈ k x]

In[1068]:= aS[n_][k_] := Simplify[∫_0^50 fp[x, -k] phiS[n][x] ⅆx, Assumptions → {n ∈ Integers}]

In[1069]:= aS[n][k]
```

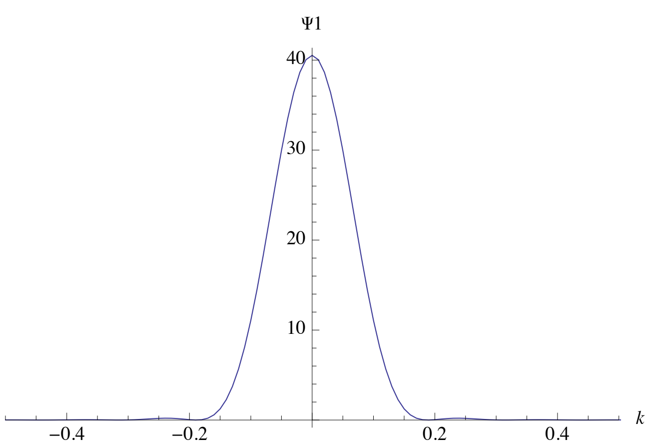
Out[1069]= $\frac{10 \left(1-(-1)^{n} e^{-50 i k}\right) n \pi}{-2500 k^{2}+n^{2} \pi^{2}}$

```
In[1070]:= coefS[n_][k_] := Simplify[aS[n][k] aS[n][-k]]

In[1071]:= coefS[n][k]
```

Out[1071]= $-\frac{100 \left(1-(-1)^{n} e^{-50 i k}\right) \left(-1+(-1)^{n} e^{50 i k}\right) n^{2} \pi^{2}}{\left(-2500 k^{2}+n^{2} \pi^{2}\right)^{2}}$

```
Plot[{Evaluate[Chop[N[coefS[1][k]]]]}, {k, -1, 1}]

$Aborted

In[1072]:= c1 = Table[{k, Evaluate[Chop[N[coefS[1][k]]]]}, {k, -1, 1, .01}];
 c5 = Table[{k, Evaluate[Chop[N[coefS[5][k]]]]}, {k, -1, 1, .01}];
 c15 = Table[{k, Evaluate[Chop[N[coefS[15][k]]]]}, {k, -1.5, 1.5, .01}];
 c101 = Table[{k, Evaluate[Chop[N[coefS[101][k]]]]}, {k, -10, 10, .1}];

π / 50.

0.0628319

In[1083]:= ListPlot[{c1}, Joined → True, PlotRange → {{-.5, .5}, All}, AxesLabel → {k, Ψ1}]
```

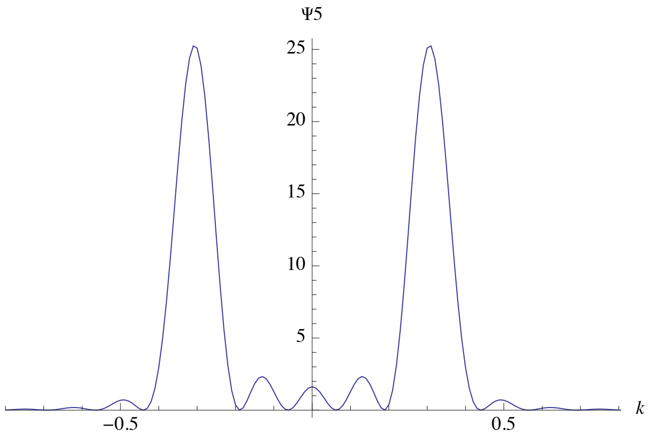
Out[1083]=




```
π 5 / 50.

0.314159
```

In[1082]:=
```
ListPlot[{c5}, Joined → True, PlotRange → {{-.8, .8}, All}, AxesLabel → {k, Ψ5}]
```

Out[1082]=

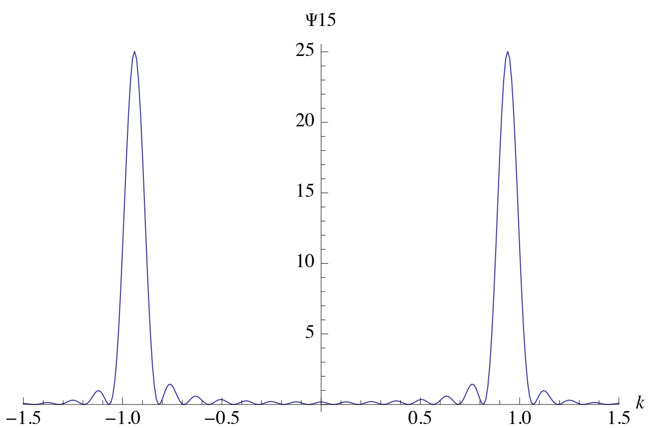



```
π 15 / 50.
```

```
0.942478
```

In[1081]:=
```
ListPlot[{c15}, Joined → True, PlotRange → {{-1.5, 1.5}, All}, AxesLabel → {k, Ψ15}]
```

Out[1081]=




```
π 101 / 50.
```

```
6.34602
```

In[1080]:=
```
ListPlot[{c101}, Joined → True, PlotRange → All, AxesLabel → {k, Ψ101}]
```

Out[1080]=